



A Fuzzy Approach to the Synthesis of Cognitive Maps for Modeling Decision Making in Complex Systems

Aslan A. Tatarkanov ^{1*}, Islam A. Alexandrov ¹, Leonid M. Chervjakov ¹,
Tatyana V. Karlova ¹

¹ IDTI RAS Institute for Design - Technological Informatics of RAS, Moscow, Russian Federation.

Abstract

The object of this study is fuzzy cognitive modeling as a means of studying semistructured socio-economic systems. The features of constructing cognitive maps, providing the ability to choose management decisions in complex semistructured socio-economic systems, are described. It is shown that further improvement of technologies necessary for developing decision support systems and their practical use is still relevant. This work aimed to improve the accuracy of cognitive modeling of semistructured systems based on a fuzzy cognitive map of structuring nonformalized situations (MSNS) with the evaluation of root-mean-square error (RMSE) and mean average squared error (MASE) coefficients. In order to achieve the goal, the following main methods were used: systems analysis methods, fuzzy logic and fuzzy sets theory postulates, theory of integral wavelet transform, correlation and autocorrelation analyses. As a result, a new methodology for constructing MSNS was proposed—a map of structuring nonformalized situations that combines the positive properties of previous fuzzy cognitive maps. The solution of modeling problems based on this methodology should increase the reliability and quality of analysis and modeling of semistructured systems and processes under uncertainty. The analysis using open datasets proved that compared to the classical ARIMA, SVR, MLP, and Fuzzy time series models, our proposed model provides better performance in terms of MASE and RMSE metrics, which confirms its advantage. Thus, it is advisable to use our proposed algorithm in the future as a mathematical basis for developing software tools for the analysis and modeling of problems in semistructured systems and processes.

Keywords:

Semistructured Systems;
Fuzzy Logic;
Fuzzy Cognitive Maps;
Cognitive Analysis;
Dynamic Models.

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1- Introduction

With the growth of human needs, many theoretical and applied tasks are solved using computer technology and various information systems, including automated DSS [1-3]. The difficulties of solving the arising problems are related not only to the need to perform a huge number of procedures, which should be done as quickly and qualitatively as possible, but also to the fact that the initial data requirements to solve them are variable and often have an uncertain, fuzzy nature. This problem is particularly acute in the medical subject area [4-6], where the uncertainty of responsible decisions may be related to a number of the following circumstances:

- Problems of complete collection and recording of information about impacts affecting the medical situation in question [7];
- Possible inaccuracy of the results of medical analyses;
- The inability to engage competent expert physicians in sparsely populated hard-to-reach areas or case of emergencies;
- The manifestation of subjective human factors;

* **CONTACT:** as.tatarkanov@yandex.ru

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- Inconsistencies in the treatment policies proposed by subject matter experts in multifactorial comorbid cases;
- In the course of scientific progress, modern medicine is mastering computerized DSS. However, many barriers prevent their mass introduction [8, 9]:
- The problems associated with the enormous amount of accumulated medical knowledge and technology, as well as their constant updating;
- The absence of open and updated banks of large impersonal clinical data available to researchers, verified by subject matter experts, and then normalized and, if necessary, generalized;
- Insufficient data formalization and standardization;
- The complexity of reasoning the decision-making methods recommended by the DSS.

Attempts to overcome such barriers in solving design problems of automated DSS in medical and other subject areas lead to new methodological approaches and modeling methods.

In the study of complex systems, as a rule, it is not possible to build a reliable mathematical model because of the large uncertainty in the interaction of system elements. Consequently, approaches using the application of specially developed decision support mechanisms based on elements of the fuzzy set theory, algebra of logic, semantic networks, the theory of cognitive analysis, and others were developed [10-12]. One method of modeling an uncertain and dynamic environment is the cognitive approach. The essence of cognitive modeling, also called “cognitive structuring” is to assist the decision-maker. Cognitive structuring makes it possible to delve into the situation and develop the most appropriate decision, relying not on intuitive choice but an ordered and verifiable knowledge of the subject area under study.

The basis of a wide range of developed cognitive models is various types of cognitive maps of the subject area under study, a set of factors describing it (commonly called concepts), where a set of cause-and-effect relations is given. It is a particular scheme that helps to consider a particular problem in any sphere of life in more detail, thoroughly examining the relations between its components. Because of constructing a cognitive map, it is possible to look at the problem or situation from the outside, understand what and how to change, and where to go better [13, 14].

In solving many practical problems of cognitive modeling and analysis of dynamic semistructured systems, the method of fuzzy cognitive maps (FCM) has quite successfully established itself [15-17]. Nevertheless, further improvement of such technologies necessary for DSS development and their practical use remains relevant. In this regard, the purpose of this paper is to improve the accuracy of cognitive modeling of semistructured systems based on fuzzy cognitive MSNS with RMSE and MASE coefficient estimation.

2- Literature Review

The practical application specifics of cognitive modeling tools focus on analyzing certain scenarios in the subject area under consideration, considering its peculiarities. This analysis aims to detect contradictions and analyze observed events and qualitative comprehension of the problems in the subject area under study. The main stage of cognitive analysis in cognitive modeling is reduced to simplifying the representation of internal processes and trends of the system and its existing problems. Cognitive modeling aims to study possible ways of developing events and solving problems arising in simulated situations. Let us consider a few examples below.

Chen and Chiu (2021) used the FCM method to illustrate the development opportunities of the Internet of Things industry in Taiwan [18]. The study combines expert opinions with learning algorithms and obtains a dynamic fuzzy model of the cognitive map to solve a multi-criteria analysis problem. FCMs work effectively in short-term forecasting and usually fail to justify themselves in long-term forecasting due to potentially complex interactions in the next steps. Feng et al. (2021) propose a robust conceptual method for long-run time series forecasting using FCM-algorithms [19]. The method efficiency is shown on both synthetic and real data. Babroudi et al. (2021) applied the FCM-algorithm in combination with Z-estimation of uncertainty to assess the causal relationship between the criteria of health care services in infectious disease outbreaks; the importance of effective criteria for their quality was evaluated concerning their interaction with each other [20].

Baykasoğlu and Gölcük (2021) propose computational methods based on alpha pruning for FCM modeling [21]. The described model is implemented in a real problem of selecting a third-party logistics service provider. Wang et al. (2021) describe an FCM method using a batch multitask evolutionary algorithm based on random inactivation that is effective on synthetic and real datasets in reconstructing the regulatory gene network [22]. Liu and Liu (2020) propose an ingenious time series prediction method based on FCM algorithms [23]. A prediction method based on empirical mode decomposition and high order fuzzy cognitive networks is applied. The increased ability for reliability growth and insensitivity to hyperparameters is demonstrated. Chen et al. (2020) consider the application of adaptive FCM for risk assessment in public-private partnership projects [24]. The proposed method can study random relationships based on observable data and assess risks associated with uncertainty, subjectivity, and complex interdependencies. Akinnuwesi et al. (2020) describe the FCM application for creating DSSs designed for the early diagnosis of rheumatoid diseases of the musculoskeletal system [25]. The method considers causal interactions between symptoms and risk factors and is

highly effective. Another study describes the use of a multi-layer FCM for the early diagnosis of autism spectrum disorders. The model has demonstrated its effectiveness on real diagnostic datasets and improved accuracy compared to traditional FCMs [26].

Thus, the cognitive analysis uses many approaches with different methodological schemes based on the peculiarities of the object (or objects) and subject area under study. Information analysis suggests an important area of research in this subject area is the study of possible approaches to improving accuracy, such as using new methods for decomposition of time series of data and signals in combination with FCM. Generalization and unification of existing approaches allow us to distinguish seven key stages inherent in the cognitive analysis process in most cases where any of them is responsible for solving a strictly definite task. The key goal achievement of cognitive analysis happens during a sequential realization of all these stages. Thus, the process of cognitive analysis may be represented in a generalized form as an algorithm with the stage-by-stage implementation of the following procedures:

- Formulating the goal and identifying the objectives of cognitive analysis. At this stage, the research object is studied, and the subject area is analyzed.
- Conducting the situation analysis for this goal. At this stage, it is necessary to collect as much complete information as possible. The most important data is systematized and analyzed, taking into account the subject area. Requirements, restrictions, and conditions for a particular situation are established.
- Cognitive modeling (initial stage) – establishing the key concepts on which the development of the situation depends. At this stage, concepts of a studied situation should be revealed and analyzed.
- Cognitive modeling – establishing the interrelationship of concepts. Causal relationships are analyzed at this stage, and a cognitive map is constructed in a functional graph.
- Cognitive modeling (final stage) – establishing the impact of various concepts on each other. At this stage, mathematical models are constructed for formalizable dependencies. For not formalizable qualitative relations between concepts, the expert's subjective opinion is used.
- Verification of the cognitive model. At this stage, the correspondence of the created model to the observed situation from the investigated subject area is evaluated.
- The resulting model testing and setting. At this stage, mechanisms and ways to ensure the necessary results in response to the situation and exclude adverse consequences are established. A management strategy is selected. Forces that drive trends are set, and target directions of the situation development are defined.

Cognitive modeling of the situation is implemented within stages 3, 4, and 5. The cognitive modeling result is presented as an oriented graph with vertices, which are factors (concepts). The arcs linking the graph vertices are interpreted as direct causal relationships (or impacts) between the factors. Thus, an essential foundation of the entire family of cognitive analysis models is constructing an oriented graph, a cognitive map describing how concepts depend on each other. Traditional cognitive maps contain relationships that take one of three possible values from the set $\{-1, 0, +1\}$. A relationship value of $+1$ and -1 between two concepts indicates the first concept's positive and negative influence on the second one. Zero indicates the absence of connecting relations between the concepts. In this case, the cognitive map does not indicate the relationship.

FCMs, unlike traditional maps, represent the fuzzy-oriented graph with feedback. Its nodes are fuzzy sets, and directed edges of FCMs reflect causal relationships between concepts and determine the degree of influence (weight) of related concepts.

In a generalized form, within algorithms for FCM construction, such procedures are performed in stages as follows:

- Definition of the list of concepts that are significant for a given subject area, then for each concept, it is necessary to define a level scale on which the concept values will be set;
- Definition of causal (influence) relationships between each pair of concepts;
- Determining the sign of influence (positive or negative) between each pair of concepts related by a causal relationship;
- Determining the power of influence between each pair of concepts related by a causal relationship;
- Determining the initial state of the concepts;
- Determining the external influences on the concepts.

Traditional cognitive maps can be used only in problems where it is necessary to assess the impact of particular concepts on the entire system's stability. The impossibility of numerical modeling of the behavior of systems described by simple cognitive maps leads to the limitation of their application in solving practical problems. At the same time, the sphere of FCM practical application created with the use of various methodological approaches, on the contrary, noticeably expands. This expansion is associated with their ability to provide the flexibility of the subject area representation, the possibility of abstract interpretation of causal relationships between concepts, and using elements of

fuzzy logic – so-called judgments. Currently, the mathematical apparatus oriented to the application of FCM is well developed. There are many algorithms for the construction and functioning of such maps, such as FCMs proposed by Kosko (1986) [27], the differential Hebbian algorithm [28], balanced differential algorithm, generalized FCMs [29], fuzzy production cognitive maps, fuzzy cognitive approach to dynamic system management, an algorithm based on the coordinated processing of expert and statistical information, and others.

The most common are Mamdani fuzzy production cognitive map (FPCM) models, in which both input and output have information given by the values of linguistic variables. The transfer of mutual influence between concepts is based on Mamdani's fuzzy inference method, where fuzzy sets condition the values of input and output variables. In order to reveal the influence of a group of input concepts on the output concept in FPCM, a special kind of operation "fuzzy accumulation with transfer" is used, which essence is the following: influences from two input concepts, defined by fuzzy sets, are represented by the membership function of the output concept. The membership function "moves" in the basic set to the area of larger values from smaller values until it reaches maximums and discretizes. Then the obtained membership values are summed up in a sequence starting from the smallest one. The sum of more than one is summed with the next point value.

In FPCM, unlike other types of cognitive maps, both "fuzzy coordinates" are implemented. However, the disadvantages of FPCM can be attributed to the fact that this mechanism itself is implemented quite arbitrarily, and there is no possibility to describe it explicitly. Different approaches (in FCM development) to define arcs, vertices, weights on arcs, and different defining functions between the factors periodically generate different modifications of cognitive map-based models and formal and mathematical apparatus changes for their analysis. A possible modification of the FCM is proposed in this paper for MSNS.

3- Methodology

A flowchart illustrating our proposed methodology is shown in Figure 1.

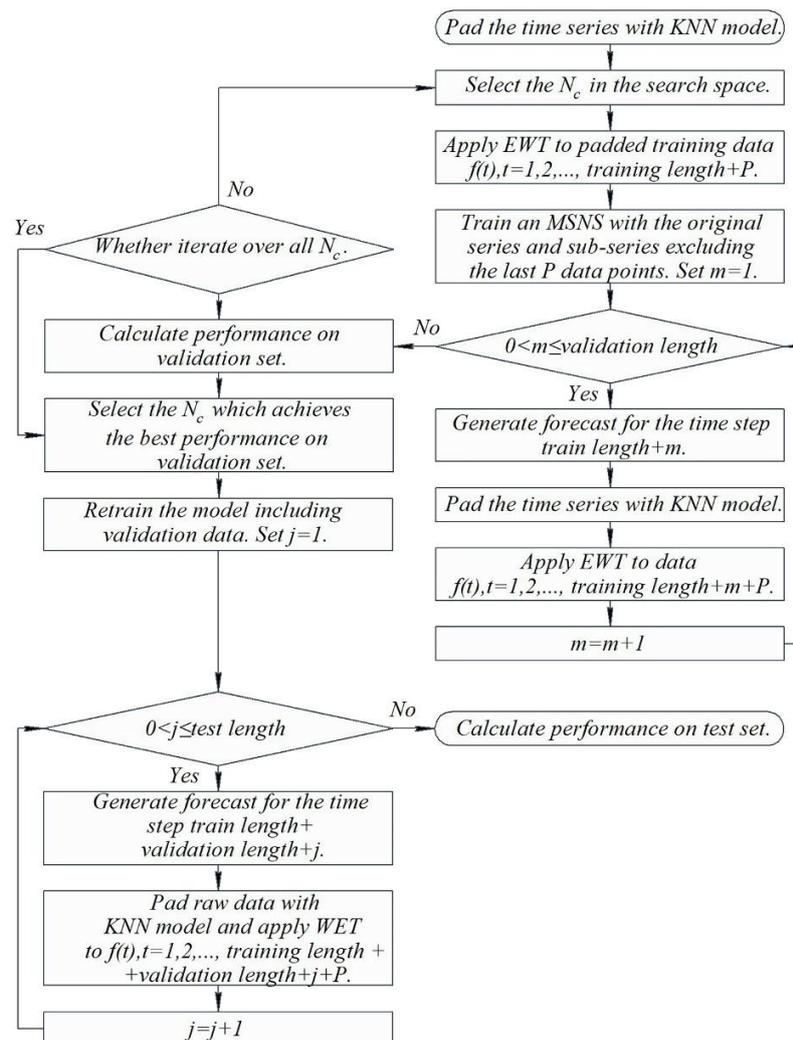


Figure 1. Flowchart of the proposed methodology

3-1- Map of Structuring Nonformalized Situations (MSNS)

A comprehensive opportunity analysis of existing methods of FCM construction made it possible to formulate some requirements whose implementation should be ensured within the development of the MSNS method.

1. In analyzing and constructing MSNS, it is necessary to maximize the mathematical apparatus of the fuzzy sets theory in the developed map. All the objects (fuzzy concepts and mapping the influence) included in MSNS and all actions performed with them (considering the influence of several concepts, dynamic modeling) should be fuzzy.
2. Fuzzy relations between concepts (mutual influence of concepts) should be represented by a fuzzy mapping of an input concept on an output concept. A fuzzy set of values represents these concepts. Influences may also be represented by functions that will be clear.
3. Fuzzy concepts describing objects of the subject area should be represented quantitatively, i.e., be formalized. It is possible to represent them as clear values, single-point belongings, or fuzzy sets within the modeled system.
4. It is necessary to implement dynamic models in the map to consider the system behavior, which will change under the mutual influence of the concepts and be non-linear.
5. In MSNS, it is necessary to consider opposite vectors of weights of mutual influence of concepts and accumulation of such influence and implement a mechanism that would simultaneously consider positive and negative influences of concepts and represent them both as fuzzy membership functions and simple values.
6. It is necessary to develop an algorithm for training MSNS on a sample of reference scenarios.
7. The accumulation of mutual influence of concepts should be implemented by a separate procedure with a cumulative, additive character to consider influences with even the smallest values.
8. The property of associativity and commutativity should be respected in this procedure of mutual influence accumulation of concepts. Its operation should not be affected by the consistency of individual factors in the subject area.
9. It is necessary to separately consider the input and output concepts and mark them when constructing MSNS.
10. For proper consideration in MSNS, it is necessary to move the fuzzy membership function in the basic set to larger values from smaller values and consider it in the mutual influence accumulation of concepts. The algorithm should permit the training of MSNS relationship weights using the generated training sample. The proposed MSNS results from the formalization of a fuzzy cognitive model based on formalizing causal relationships between multiple factors (concepts) characterizing the system under study. The proposed MSNS structure based on these requirements is shown in Figure 2.

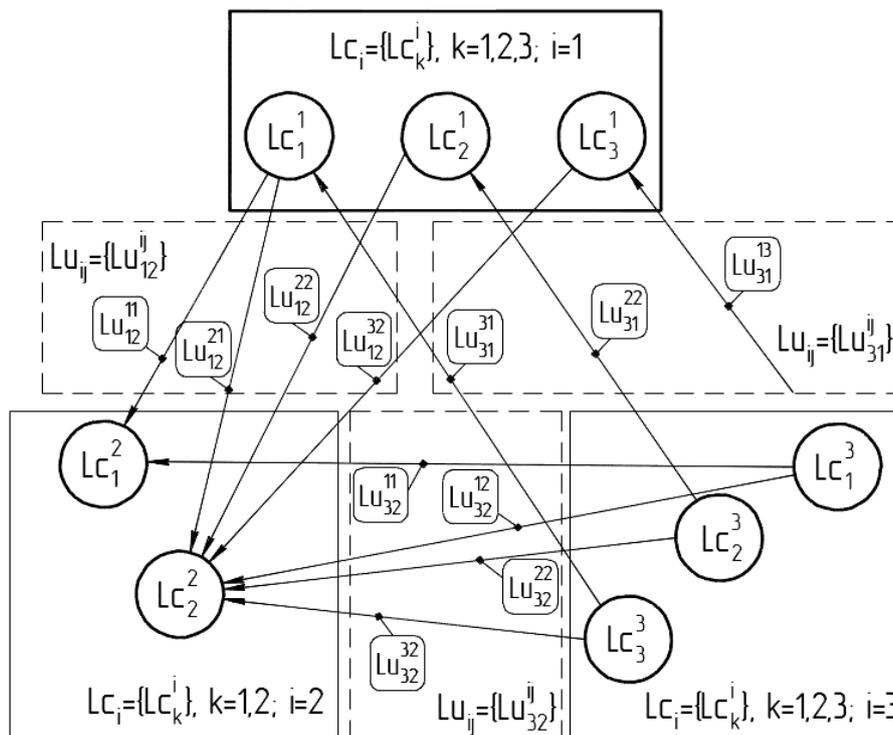


Figure 2. MSNS Structure

All designations of variables and relationships indicated in Figure 2 are presented in the text of the article below. We consider MSNS as a fuzzy network of mutual influence of concepts as follows (1):

$$MSNS = (C, U) \quad (1)$$

where C is the set of concepts defined by the tuple (2):

$$C = \{C_i\} \text{ at } (i = 1, 2, \dots, n) \quad (2)$$

where n is power of a set of concepts; U is the set of relationships between concepts (3)

$$U = \{u_{ij}\} \text{ at } (j = 1, 2, \dots, n) \quad (3)$$

A cognitive map is a causal network reflecting some area of knowledge through the nodes and arcs of the semantic network of the subject area, which determines the proximity of the theoretical aspects of MSNS and semantic networks construction. Each particular concept C_i may be represented by LP_{C_i} -the linguistic variable defined by the following set (4):

$$LP_{C_i} = \langle \tilde{C}_i, L_{C_i}, P_{C_i} \rangle \quad (4)$$

where P_{C_i} is base term set \tilde{C}_i ; L_{C_i} is the list of linguistic values of the i -th concept of the subject area, which is given by the set of basic states characterizing it (5):

$$L_{C_i} = \{L_{C_k^i}\} \text{ at } (k = 1, 2, \dots, m_i) \quad (5)$$

where m_i is the number of base states of the i -th concept. Each set element $L_{C_k^i}$ is its term that includes the basis (state) of the concept C_i represented by a triple of fuzzy variable: $\langle L_{C_i}, P_{C_i}, \tilde{S}_{C_k^i} \rangle$. $\tilde{S}_{C_k^i}$ is a fuzzy set in the base set P_{C_i} , which an expression of the form may represent:

$$\tilde{S}_{C_k^i} = \{p, \mu_{\tilde{S}_{C_k^i}}(p) | p \in P_{C_i}\} \quad (6)$$

Where $\mu_{\tilde{S}_{C_k^i}}(p)$ is fuzzy membership function; p is element belonging to the base set P_{C_i} . The relationships between concepts (elements of the set U) determine the degree of influence between the subject area concepts. A specific value also determines them from the term set:

$$LP_{u_{ij}} = \langle \tilde{U}_{ij}, Lu_{ij}, Pu_{ij} \rangle \quad (7)$$

where Pu_{ij} is base term set \tilde{U}_{ij} ; Lu_{ij} is list of linguistic values of the degree of relationship between the i -th and the j -th concepts of the subject area, which characterizes the set of its basic states and is given by the expression:

$$Lu_{ij} = \{Lu_{kh}^{ij}\} \text{ at } (h = 1, 2, \dots, m_j) \quad (8)$$

where m_j is the number of such states in the j -th concept; $k \times h$ is the number of values. Each set element Lu_{kh}^{ij} is a term containing the basis (state) of influence weight \tilde{U}_{ij} between pairs of i -th and j -th MSNS concepts represented by a triple of fuzzy variables $\langle Lu_{kh}^{ij}, Pu_{ij}, \tilde{S}_{u_{kh}^{ij}} \rangle$. $\tilde{S}_{u_{kh}^{ij}}$ is fuzzy set in the base set Pu_{ij} , which is written by an expression of the form:

$$\tilde{S}_{u_{kh}^{ij}} = \{p, \mu_{\tilde{S}_{u_{kh}^{ij}}}(p) | p \in Pu_{ij}\}$$

where $\mu_{\tilde{S}_{u_{kh}^{ij}}}(p)$ is fuzzy membership function, Pu_{ij} ($i = 1..n$; $j = 1..n$); n is the power of the set of the subject area concepts.

MSNS uses the same mechanism as FPCM algorithms for a combined view of positive and negative interrelationships of concepts and the accumulation of these influences. For this purpose, extending the base set of concept states and the set of interrelationship weights from negative values is necessary. Then it is necessary to apply the productive systems of the Mamdani type where the mutual influence transfer between concepts is based on fuzzy inference with the values of input and output variables conditioned by fuzzy sets. The Mamdani algorithm was used to obtain fuzzy inference according to the following scheme:

$$U_{p=1}^{kj} (\cap_{i=1}^n x_i = a_{i,jp} \text{ with weight } u_{jp}) \rightarrow y = d_j, \quad j = \overline{1..m} \quad (10)$$

where fuzzy sets define the values of input and output concepts. Note that expression (10) is quite formal since it does not contain specific fuzzy rules for transitioning from input to output variables. Thus, it is necessary to introduce a rule base, denoting which membership functions of fuzzy variables are used.

In practice, many different types of membership functions are used in the theory of fuzzy sets. Let us list some common types of membership functions with their advantages and disadvantages:

- Piecewise linear (multiangular) membership functions that are not continuously differentiable;
- Intuitive symmetric Gaussian function in which obtaining simple local linear response surfaces is difficult since unity partition conditions are violated;
- Intuitive asymmetric Gaussian function which deprived of the mentioned disadvantages of the symmetric Gaussian function;
- Sigmoidal membership function, which, unlike Gaussian functions, is suitable for representation of extreme sets;
- High-order polynomial membership function which combines the possibility of a significant increase in accuracy with the high complexity of finding a large number of parameters required to define the membership function;
- Harmonic membership function in which infinite differentiability simplifies obtaining smooth, continuously differentiable model response surfaces.

Many conversion functions are available for the FCM. When the state values are in the range [-1, 1], it is necessary to use the hyperbolic tangent function (11):

$$\tanh(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \tag{11}$$

According to Formula 11, FCMs can model only short-term temporal relationships, and the process of modeling long-term temporal dependencies was proposed by Stach et al. (2006) [30] and is described as follows (12):

$$A_i(t + 1) = g[\sum_{j=1}^{N_c} \omega_{ij}^1 A_j(t) + \omega_{ij}^2 A_j(t - 1) + \dots + \omega_{ij}^h A_j(t - h + 1) + \omega_{i0}] \tag{12}$$

where ω_{ij}^h is the impact of node j on node i at time step $(t-h+1)$; ω_{i0} is displacement term.

The choice of one or another function is largely determined by the available information about the modeled system and the quality of the available model adjustment procedures. At the same time, the analyzed information about the system may not be equivalent, so it also needs to be evaluated. For this purpose, an empirical wavelet transform (EWT) can be used, which analyzes the signal after a fast Fourier transform and implements spectrum separation by bandpass filtering using a special set of filters based on the nature of the data. This method of adaptive signal decomposition has high efficiency in analyzing nonstationary time series and signal processing; thus, it has an effective use [31-34]. The EWT uses the Littlewood-Paley and Meyer wavelets [35] because of the convenience of closed-form expression in the Fourier domain. Construction of these bandpass filters is expressed by Equations 13 and 14, in which the transition band is controlled by a transition zone parameter γ satisfying the condition $\gamma \leq \min_n [(\omega_{n+1} - \omega_n) / (\omega_{n+1} + \omega_n)]$ to avoid spectrum overlapping [36]:

$$\hat{\Phi}_n(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq (1 - \gamma)\omega_n \\ \cos \left[\frac{\pi}{2} \beta \left(\frac{1}{2\gamma\omega_n} (|\omega| - (1 - \gamma)\omega_n) \right) \right] & \text{if } (1 - \gamma)\omega_n \leq |\omega| \leq (1 + \gamma)\omega_n \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$$\hat{\Psi}_n(\omega) = \begin{cases} 1 & \text{if } (1 + \gamma)\omega_n \leq |\omega| \leq (1 - \gamma)\omega_{n+1} \\ \cos \left[\frac{\pi}{2} \beta \left(\frac{1}{2\gamma\omega_n} (|\omega| - (1 - \gamma)\omega_n) \right) \right] & \text{if } (1 - \gamma)\omega_{n+1} \leq |\omega| \leq (1 + \gamma)\omega_{n+1} \\ \sin \left[\frac{\pi}{2} \beta \left(\frac{1}{2\gamma\omega_n} (|\omega| - (1 - \gamma)\omega_n) \right) \right] & \text{if } (1 - \gamma)\omega_n \leq |\omega| \leq (1 + \gamma)\omega_n \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

In Equations 13 and 14, the function $\beta(x)$ has the form (15) as a consequence of the fact that Equation 15 must be satisfied:

$$\beta(x) = x^4(35 - 84x + 70x^2 - 20x^3) \tag{15}$$

The graphical interpretation of Functions 13 and 14 is presented in Figure 3. The midpoints between frequencies are sorted by order and denoted as $\omega_n(1 \leq n \leq N)$. The set of constructed empirical scaling functions and wavelet

functions $\{\hat{\phi}_1(t), \hat{\psi}_n(t)_{n=1}^N\}$, according to functions 13 and 14, is identified as a rigid frame $L^2(\mathbb{R})$ together with the coefficient $\gamma \leq \min_n [(\omega_{n+1} - \omega_n) / (\omega_{n+1} + \omega_n)]$ resulting in Expression 16:

$$\sum_{k=-\infty}^{\infty} [|\hat{\phi}_1(\omega + 2k\pi)|^2 + \sum_{n=1}^N |\hat{\psi}_n(\omega + 2k\pi)|^2] = 1 \tag{16}$$

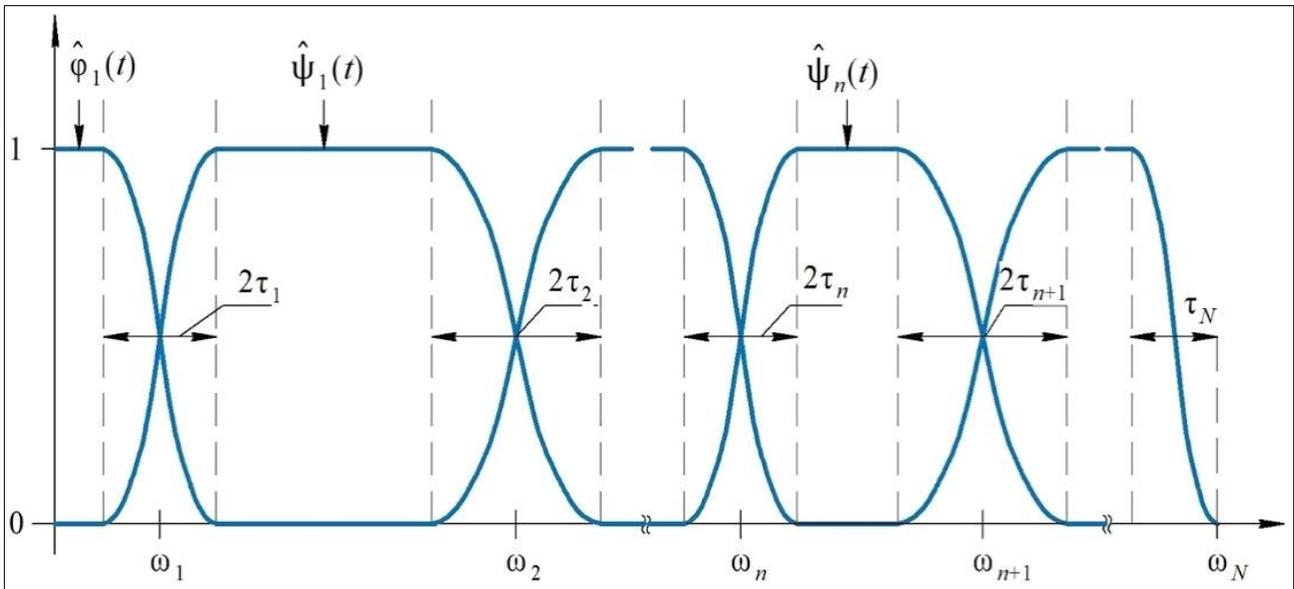


Figure 3. Graphical Representation of Functions 13 and 14

For the transition from the fuzzy set defined by the universal set of fuzzy terms (the universal set is this term-menu of the output concept y) to the fuzzy set defined on the interval, it is necessary to perform the steps listed below:

- 1) "Constrain" the membership functions of the output concept;
- 2) Perform the combining operation for the obtained fuzzy sets mathematically written through the aggregate operation of fuzzy sets:

$$\tilde{y} = a \underset{j=1,m}{\text{gg}} \left(\int_{\underline{y}_i}^{\bar{y}_i} \min(\mu_{d_j}(X^*), \mu_{d_j}(y)) / y \right) \tag{17}$$

The aggregate operation is implemented using the operation of finding the maximum. For finding the exact value of the output concept y , defined according to the input vector X , which is the result of the fuzzy set defuzzification y , the center of gravity method is used:

$$y = \int_{\underline{y}}^{\bar{y}} y \cdot \mu_{\tilde{y}}(y) dy / \int_{\underline{y}}^{\bar{y}} \mu_{\tilde{y}}(y) dy \tag{18}$$

3-2- Dynamics of MSNS Changes

Within the dynamic model implementation, MSNS should be considered, in contrast to (1), as a fuzzy network of mutual influence of concepts of the form:

$$MSNS = (C, U, F_c, F) \tag{19}$$

where C, U is elements defined earlier: (1) – (3); F_c is a function of the activity of the system's concepts; F is transfer function of the system. The function of the activity of the system's concepts may be presented as follows:

$$F_c: C_i \rightarrow F_{c_i} \tag{20}$$

Let us assign a measure of activity to each node at time t . The function can take the value "0", i.e., no activity, or "1" – active. Then $F_c(t)$ can be considered an activity (states) vector of concepts at iteration t . $F_c(0)$ will be a vector of initial values of activity of concepts. The system transfer function F may be presented as follows:

$$F: R \rightarrow L \tag{21}$$

where L is transfer function $F_{c_i}(t + 1)$ and $F_c(t)$, $t \geq 0$, that may be represented as Equation 22:

$$F_{c_i}(t + 1) = f \left(\sum_{j \neq i}^n u_{ij} F_{c_j}(t) \right) \tag{22}$$

This function is necessary to limit the sum to the range [0; 1] and may be:

– Discrete bivalent:

$$f(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \quad (23)$$

– Discrete trivalent:

$$f(x) = \begin{cases} -1, & x \leq 0.5 \\ 0, & -0.5 < x < 0.5 \\ 1, & x \geq 0.5 \end{cases} \quad (24)$$

– Continuous (logistical):

$$f(x) = \frac{1}{1+e^{-Fcx}} \quad (25)$$

This representation of MSNS over time results in a sequence of state vectors that can define the system state under simulation within further iterations. Thus, the significance of each concept can be analyzed. The result of conducting such a simulation will be the level of significance of the concept after some time interval. Setting different initialization vectors provides different simulation results.

4- Results and Implications

The section presents the testing results of the approach proposed to construct a fuzzy cognitive MSNS. The modeling results are presented compared to the initial data; error rates, types of normalization, and the crosscheck procedure are described.

Four public datasets were used to validate the benefits of our proposed approaches. Three of these datasets are time series of electricity load data for the Australian energy market [37], containing 48 electricity load measurement points for each measurement day. Time series data for New South Wales (NSW), South Australia (SA), and Victoria (VIC) were used for modeling. The fourth set of independent public data obtained from the Federal Reserve System website [38] contains information on the inventory-to-sales ratio (ISR).

Next, x data points obtained using the EWT were normalized to the x_{NORM} value in the range [-1; 1]. The maximum x_{MAX} and minimum x_{MIN} values of each time series were used for normalization by Formula 26, respectively. The choice of this normalization method is since the hyperbolic tangent Function 11 is used as a non-linear transformation function.

$$x_{NORM} = 2 \frac{x - x_{MIN}}{x_{MAX} - x_{MIN}} - 1 \quad (26)$$

Then each dataset was divided into three samples in the proportion of 70%, 20%, and 10% of the dataset. The largest sample was used as a training sample, and the other two samples were used for validation and testing. In order to prevent overtraining of the predictive model on the training set, the hyperparameters that provide the best performance on the validation data sample were selected. At the same time, metrics such as RMSE and MASE, proposed by Hyndman and Koehler (2006) [39], were used to check the performance. The RMSE metric was calculated by the formula 27, and the MASE metric by the formula 28, respectively:

$$RMSE = \sqrt{\frac{1}{L} \sum_{j=1}^L (\hat{x}_j - x_j)^2} \quad (27)$$

$$MASE = \text{mean} \left(\frac{|\hat{x}_j - x_j|}{\frac{1}{T-1} \sum_{t=2}^T |x_t - x_{t-1}|} \right) \quad (28)$$

where x_j and \hat{x}_j is initial and predicted data points; L is test sample size; T is training sample size.

Let us describe the modeling process and the results on the example of the ISR time series. Above all, the first difference in the initial time series is calculated, and then the data are subject to normalization according to the Formula 26. In order to provide cross-validation, the time series data are divided into three samples of size 70%, 20%, and 10% for training, validation, and testing, respectively. The number of nodes is selected by cross-validation using grid search.

According to the autocorrelation function (ACF) of the ISR time series presented in Figure 4, the second-highest value after zero is the value at time lag 12. The EWT is applied for decomposing the augmented time series, and then the generated data samples are used for modeling, excluding the last filled data points.

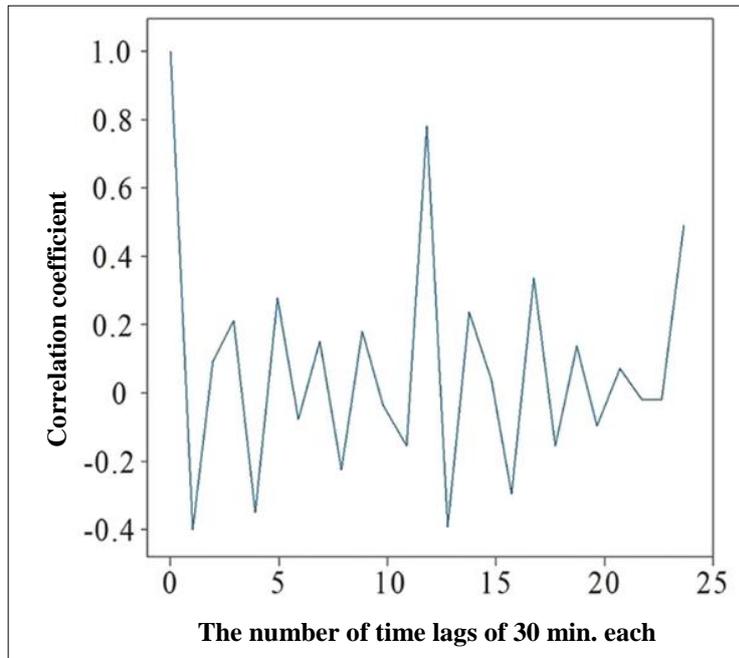


Figure 4. Autocorrelation Function (ACF) of the ISR Time Series

After defining the MSNS structure, the regularization coefficient of the ϵ -support vector regression (ϵ -SVR) is determined for each node. A special method of MSNS training is used to improve immunity to outliers based on the ϵ -SVR in which the insensitive loss function penalizes errors outside the cube but ignores errors inside it [40]. This learning process implements optimization of the MSNS weights, and then the model can be used for prediction. The well-known k-nearest neighbor (KNN) method is used to generate 12 prediction values. The time series is then decomposed by additional data points using the EWT. As a result, the trained MSNS can make predictions one step ahead until the predictions for the whole test sample have been generated. However, optimal settings should be considered hyperparameter values that ensure minimum RMSE on the validation dataset. An illustration of the results for the ISR data is presented in Figure 5, where we can see that the obtained approximation correctly reflects seasonal data fluctuations.

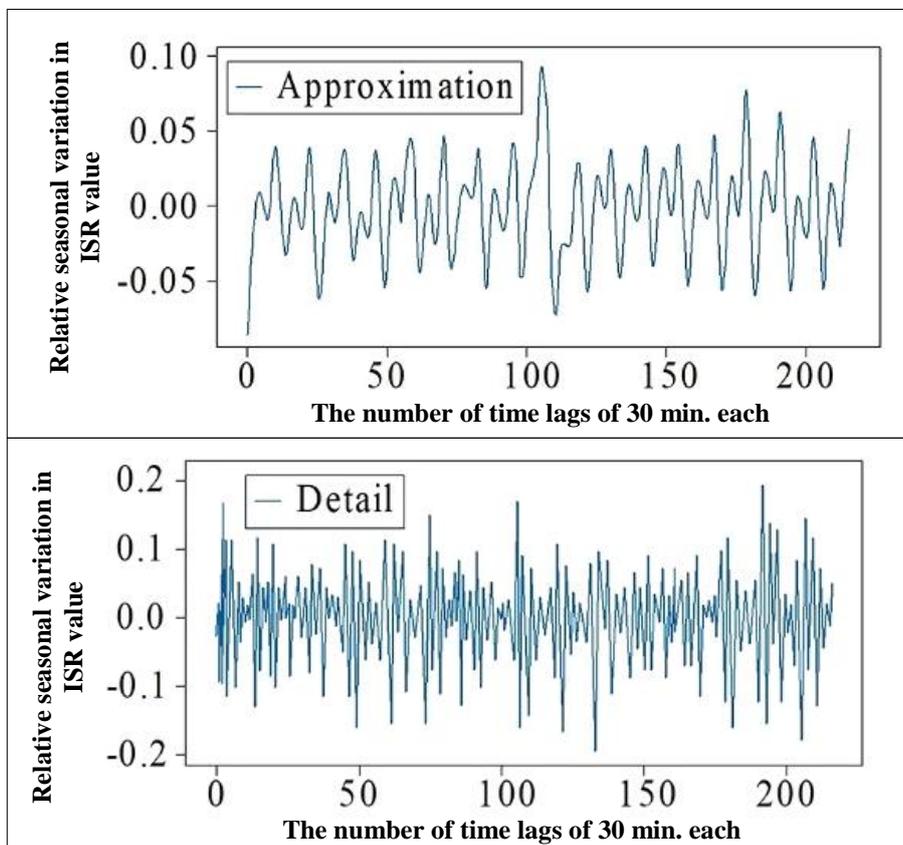


Figure 5. Approximation Data Generated Using EWT

In order to identify the advantages and disadvantages of our proposed methodology for cognitive modeling of semistructured systems based on the construction of fuzzy cognitive MSNS, we further compared its results with the results of other widely used prediction models. In addition to the ISR dataset, three previously mentioned open datasets, NSW, SA, VIC, were compared. The following models were chosen as classical models for comparison: ARIMA model [41-43], Fuzzy time series model [44-46], SVR [47], Multi-layer Perceptron (MLP) [48-50]. The learning stop criterion $\epsilon - SVR$, as proposed in [51], is controlled by error tolerance at the 1×10^{-3} level. The comparison based on RMSE and MASE metrics is presented in Table 1.

Table 1. Comparison with Classical Models by RMSE and MASE Criteria

| Dataset | Model | | | | | | | |
|-------------------|---------|--------|---------|-------|-------|-------|-------|-------|
| | NSW | SA | VIC | ISR | NSW | SA | VIC | ISR |
| | RMSE | | | | MASE | | | |
| ARIMA | 151.288 | 50.255 | 103.116 | 0.163 | 0.660 | 0.703 | 0.670 | 1.295 |
| SVR | 103.771 | 38.595 | 89.469 | 0.052 | 0.410 | 0.580 | 0.596 | 0.437 |
| MLP | 141.013 | 36.195 | 88.103 | 0.054 | 0.565 | 0.587 | 0.601 | 0.416 |
| Fuzzy time series | 133.520 | 43.379 | 108.045 | 0.107 | 0.560 | 0.676 | 0.757 | 0.852 |
| Present work | 97.702 | 33.346 | 86.185 | 0.049 | 0.392 | 0.542 | 0.572 | 0.360 |

The analysis of the results presented in Table 1 allows us to conclude that, among the models considered, our model provides the best performance in terms of RMSE and MASE for most time series. In contrast, the ARIMA model works the worst, which is explained by its linear structure. In contrast to the RMSE criterion, the MASE criterion can be used to compare prediction performance for completely different data [39]; in this regard, the statistics data for the MASE criterion are presented further in Table 2. It shows that our model achieves the lowest mean, median, minimum, and maximum MASE values. Therefore, we can conclude that our model provides the best performance on the studied datasets, compared with the classical models ARIMA, SVR, MLP, and Fuzzy time series. Thus, it can be concluded that the goal of this work to improve the accuracy of cognitive modeling of semistructured systems based on the fuzzy cognitive MSNS has been achieved. According to the experiments, we conclude that the small decomposition levels work well on time series. The prediction accuracy degenerates with the increase of decomposition level. A small decomposition level also connotes a small number of nodes and high interpretability. In addition, the proposed model achieves the best performance in terms of MASE and RMSE on these eight datasets, which validates the proposed model's superiority compared with the baseline models

Table 2. MASE Criterion Statistics

| Dataset | Model | | | | |
|---------|--------|--------|--------|-------------------|--------------|
| | ARIMA | SVR | MLP | Fuzzy time series | Present work |
| Maximum | 2.471 | 0.596 | 0.601 | 2.021 | 0.572 |
| Maximum | 0.66 | 0.348 | 0.358 | 0.56 | 0.324 |
| Median | 1.058 | 0.4555 | 0.5305 | 0.8045 | 0.392 |
| Mean | 1.1925 | 0.4753 | 0.51 | 0.9436 | 0.4265 |

5- Conclusion

Management decision-making with large amounts of information can often be associated with complexity and a lack of structured data. Compared to the choice between alternative management decisions, a higher priority has the task of preparing the management decisions themselves, which is only possible based on an in-depth analysis of the behavior and structure of a complex system. Understanding the system under study and identifying real problems, and the causes of their emergence is a prerequisite for making a good managerial decision. Based on this, a very urgent task is the development and improvement of decision support systems.

In DSS development, a proven approach is to build a cognitive model based on the FCM, a universal tool for modeling and research processes in semistructured and weakly formalized systems. Their usage makes it possible to structure the decision-making process, identify and consider the influence of significant factors, and perform modeling of the behavior of complex systems related to the task to be solved. This study proposes a new algorithm for constructing a fuzzy cognitive MSNS. This map combines the positive properties of the previously existing FCM. Based on the MSNS, modeling problems should increase the reliability and quality of analysis and modeling of semistructured systems and processes under uncertainty. It was proved using open datasets that, in contrast to the classical ARIMA, SVR, MLP, and Fuzzy time series models, our proposed model provides the best performance in terms of MASE and RMSE metrics that confirm its advantage. The comparison with the baseline models also demonstrates that the proposed algorithm can

improve generalization ability. Thus, it is advisable to use our proposed algorithm in the future as a mathematical basis for the development of software tools for the analysis and modeling of the problems of semistructured systems and processes. A possible direction for future research could be the improvement of time series decomposition methods in conjunction with FCMs.

6- Declarations

6-1- Author Contributions

Conceptualization, A.T., and I.A.; methodology, L.Ch.; software, T.K.; validation, L.Ch., T.K., and I.A.; formal analysis, I.A.; investigation, T.K.; resources, I.A.; data curation, A.T.; writing—original draft preparation, L.Ch.; writing—review and editing, T.K.; visualization, I.A.; supervision, A.T.; project administration, A.T.; funding acquisition, L.Ch. All authors have read and agreed to the published version of the manuscript.

6-2- Data Availability Statement

Data sharing is not applicable to this article.

6-3- Funding

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6-4- Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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