Available online at www.ijournalse.org

**Emerging Science Journal**

(ISSN: 2610-9182)

Vol. 6, No. 2, April, 2022

Review Article

# A Review on Neutrino Oscillation Probabilities and Sterile Neutrinos

Kendra Jean-Jacques [1], Anna Roland [1], Christelle Billan [1], Preet Sharma [1*]

[1] *Department of Chemistry & Physics, Midwestern State University, Texas, United States.*

**Abstract**

In the past decades, there have been many groundbreaking discoveries and advancements in the field of particle physics. One of the important elementary breakthroughs is the phenomenology of neutrino oscillations. This includes the properties of neutrinos in the Standard Model (SM) and how neutrino oscillations and their properties have been so important in strengthening the SM. Neutrino oscillations also play a vital role in understanding the current nature of our Universe and the way it behaves. There is also a great interest in neutrino oscillations and their connection with dark matter. In this review, we start with the introduction and discuss the theoretical background of neutrino oscillations and some experiments, which are working to detect the properties of neutrinos. Then the fundamentals of neutrino oscillations and their interactions were described. Since there are multiple sources of neutrinos, we have described the three sectors through which we can expect neutrinos to be produced. These are the atmospheric, solar, and reactor sectors. A brief section on the important milestones in neutrino oscillations is included because of the experiments and what they use to detect neutrino properties. Finally, we also include a section on sterile neutrinos since they have been under study for a long time and there is a possibility of them being connected to dark matter interactions.

**Keywords:**

Neutrino Oscillation Probabilities;
Standard Model;
Particle Physics;
Quantum Field Theory.

**Article History:**

| | | | |
|---|---|---|---|
| **Received:** | 04 | September | 2021 |
| **Revised:** | 15 | December | 2022 |
| **Accepted:** | 11 | January | 2022 |
| **Available online:** | 09 | March | 2022 |

## 1- Introduction

Neutrino experiments have shown that neutrinos change flavor as they travel from one point to another. This phenomenon and property of neutrinos is commonly known as neutrino oscillations. Earlier, the Standard Model of Particle Physics (SM) suggested that neutrinos did not have mass. However, it was seen experimentally that neutrino oscillations can only occur if neutrinos have mass [1, 2]. Despite all the understanding, neutrino physics is still far from being completely understood. This understanding and discovery of the phenomenon of neutrino oscillations gave us a deeper insight into building the SM. The initial concept of neutrino oscillations was suggested by Pontecorvo [3, 4].

The first experimental evidence of neutrino oscillations was seen in the atmospheric Super-Kamiokande [5], the solar SNO [6] and the reactor KamLAND [7]. These experimental evidences gave us an insight which provided opportunities to look beyond SM physics. Despite decades of studies, there are still many questions, which we have not answered. Some topics which still need a deeper understanding are, oscillation parameters, neutrino masses, hierarchy, charge-parity cp-violation in the leptonic sector [8, 9]. Some very challenging questions that are of great interest to theorists and experimentalists alike are questions related to cp-violation. The currently accepted neutrino model, known as Mikheyev-Smirnov-Wolfenstein (MWS) model applies accurately to solar neutrino oscillations in vacuum and in matter independently. But, this does not apply to the transition of neutrino oscillations when neutrinos travel from vacuum to matter [9]. There is still no consensus on this transition and how it happens. Also, we still do not understand exactly what happens exactly in the transition region. Some ideas and theories about this transition are studied in the light of

* **CONTACT**: preet.sharma@msutexas.edu

**DOI:** http://dx.doi.org/10.28991/ESJ-2022-06-02-015

non-standard neutrino interactions, and these theories suggest the existence of a light sterile neutrino. There are other suggestions and studies that point to neutrino models beyond SM or new physics scenarios [10]. It is well known that in the SM, neutrinos interact with matter through weak interactions. This means that it does not contain the data from the QCD aspects [8]. There are various ways this can be approached. Some of these approaches are neutrino oscillation probability disappearance and appearance events, long baseline detectors such as LBNL, and measurements of neutrinos from various sources such as the sun, nuclear reactors, or the atmosphere [8, 11].

The electroweak theory formulated by Glashow, Weinberg and Salam, which was a theory of conserved lepton flavor, predicted that neutrinos were massless. This theory suggested that a neutrino of flavor electron (e), tau ($\tau$) or muon ($\mu$) produced in the Charged Current (CC) neutrino interactions resulting in a charged lepton flavor would not change its flavor [1]. Many observations have shown that the flavor of neutrinos change into one another while propagating large distances. The solar neutrino anomaly [12] and atmospheric neutrino anomaly [13] were two main observations that concluded that the lepton flavor violation was also possible. This conclusion was also confirmed by observation the flavor violation of man-made neutrinos [10, 13-15] and in long baseline experiments [16, 17]. The conclusions from experiments which showed lepton flavor violation made us understand the masses of the neutrinos and the phenomena of neutrino mass mixing. According to this mas mixing phenomena, each neutrino flavor is a mixture of various mass eigenstates [10, 18]. As neutrinos propagate, each component of the mass eigenstate results in a different phase, thus a neutrino of certain flavor will convert to a mixture of different flavors. Hence, the lepton flavor violation takes place [10]. In the framework of this mass mixing scheme, the probability of converting from $v_\alpha$ to $v_\beta$, where $\alpha$ and $\beta$ are the flavors, in vacuum or in matter is time dependent or we can also say that it is dependent on the distance traveled by neutrinos [19-21]. This is the reason that the converting of neutrinos from one flavor to another is generally known as neutrino oscillation. Neutrino flavor eigenstates are usually denoted by $v_\alpha$. Here $v_\alpha$ is defined as the state which is seen in a process which occurs involving a W boson vertex along with a charged lepton. Neutrino mass eigenstates are denoted by $v_i$ with masses $m_i$ where $i = 1,2,3$. The flavor eigenstates are connected to mass eigenstate by a $3 \times 3$ matrix, U, known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$v_\alpha = \sum_i \; U_{\alpha i} V_i \tag{1}$$

This $3 \times 3$ matrix U is unitary. The unitary mixing matrix can be written as,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_{23} & sin\theta_{23} \\ 0 & -sin\theta_{23} & cos\theta_{23} \end{pmatrix} \begin{pmatrix} cos\theta_{13} & 0 & sin\theta_{13}e^{-i\delta} \\ 0 & & 1 \\ -sin\theta_{13}e^{i\delta} & 0 & cos\theta_{13} \end{pmatrix} \begin{pmatrix} cos\theta_{12} & sin\theta_{12} & 0 \\ -sin\theta_{12} & cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where $\theta_{ij}$ are known as the mixing angles and are expected to be in the range of $(0, \frac{\pi}{2})$ and $\delta$ is known as the phase and it has a range of $(0, \frac{\pi}{2})$ [20]. We know that $v_1$, $v_2$, and $v_3$ have been defined according to their contribution to $v_e$. They can also be written as:

$$|U_{e1}| > |U_{e2}| > |U_{e3}| \tag{3}$$

So $v_1$ has the largest contribution and $(v_3)$ provides the smallest contribution to $v_e$. As mentioned in [19] we know that:

$$\theta_{12}, \theta_{13} \leq \frac{\pi}{4} \tag{4}$$

We can say if $v_i$ is the lightest or the heaviest. Also, we know that:

$$\Delta m_{ij}^2 = m_i^2 - m_j^2 \tag{5}$$

The study in the article [20] explains that the time evolution of ultra- realistic neutrinos is given by the Hamiltonian,

$$H = H_{vacuum} + H_{matter} \tag{6}$$

where $H_{vacuum}$ is the Hamiltonian in vacuum and $H_{matter}$ is the Hamiltonian in matter. The effective Hamiltonian in vacuum is given by:

$$H_{vacuum} = U \cdot (Diag[\frac{m_1^2}{2E}, \frac{m_2^2}{2E}, \frac{m_3^2}{2E}]) \cdot U^\dagger \tag{7}$$

Within the Standard Model (SM) of particle physics, the effective Hamiltonian in matter, $H_{matter}$, which includes the properties of the medium in which neutrinos travel is written as:

$$H_m = \begin{pmatrix} \sqrt{2}G_F N_e - \frac{\sqrt{2}}{2} G_F N_n & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} G_F N_n & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{2} G_F N_n \end{pmatrix} \tag{8}$$

Some assumptions are that the medium is electrically neutral, un-polarized and composed of non-relativistic particles. In vacuum, the Hamiltonian is $H_m = 0$, and we can write:

$$P(v_\alpha \to v_\beta) = |\sum_{ij} U_{\alpha i} U^*_{\beta j} \, e^{i\Delta m^2_{ij}\frac{L}{2E}}|^2 \tag{9}$$

Applying trivial matrix algebra techniques using the identity matrix and the Hamiltonian, neutrinos get a phase angle [20]. This is why the neutrino oscillation probabilities are dependent mainly on $m^2_{ij}$ . Another interesting and challenging aspect is CP-violation. CP violation in neutrino sector is defined by the Jarlskog invariant which is given as:

$$J = \sin(\theta_{13})cos^2(\theta_{13})sin(\theta_{12})cos(\theta_{12})cos(\theta_{23})sin(\theta_{23})sin(\delta) \tag{10}$$

The values of the mixing angles $\theta_{12}$ , $\theta_{13}$ and $\theta_{23}$ are obtained from experiments and observations. The mixing angle $\theta_{23}$ has a value which is close to 45° but it is not clear within current understandable uncertainties if we have $\theta_{23} < \frac{\pi}{4}$ or $\theta_{23} > \frac{\pi}{4}$. This uncertainty is known as the octant degeneracy. The value of the phase, $\delta$, is unknown, However, experimental data suggests a preferred value close to $(\frac{3\pi}{2})$ [10, 18]. The absolute value of $\Delta m^2_{21}$ are determined in the study. While $|\Delta m^2_{31}|$ is measured, the sign of $\Delta m^2_{31}$ is not understood completely. If $\Delta m^2_{31} < 0$ the scheme is called normal ordering or normal mass spectrum, and if $\Delta m^2_{31} < 0$, the scheme is called inverted ordering or inverted mass spectrum [10, 18]. The aim of current and future neutrino oscillation experiments are to determine $(\cos 2\theta_{23})$, $(\Delta m^2_{31})$ and the value of the CP-Violating phase $\delta$ [20].

There are other neutrino experiments [22] which measure the energy-dependent charged-current cross section of the neutrinos for the muon-neutrinos. The neutrino charge current interactions are of extreme importance because they may open the window to understanding new physics interactions which have not been observed before. Some studies [23-25] and experiments such as MiniBooNE are looking into this search too. Charge current interactions might also give us a view on cp-violation understanding. There may be a good possibility of explaining the cp-violation problem through these interactions. Other studies involving the charge current interactions in understanding the cross-section of the neutrino resulting in other particles are also underway at the MiniBooNE [26-28]. This review consists of two sections and is organized as follows. In section 2, the theoretical aspects of neutrino experiments and probability calculations are reviewed. The various types of neutrino sectors are further discussed.

## 2- Methodology

The flowchart methodology of this study is available in the Figure 1.

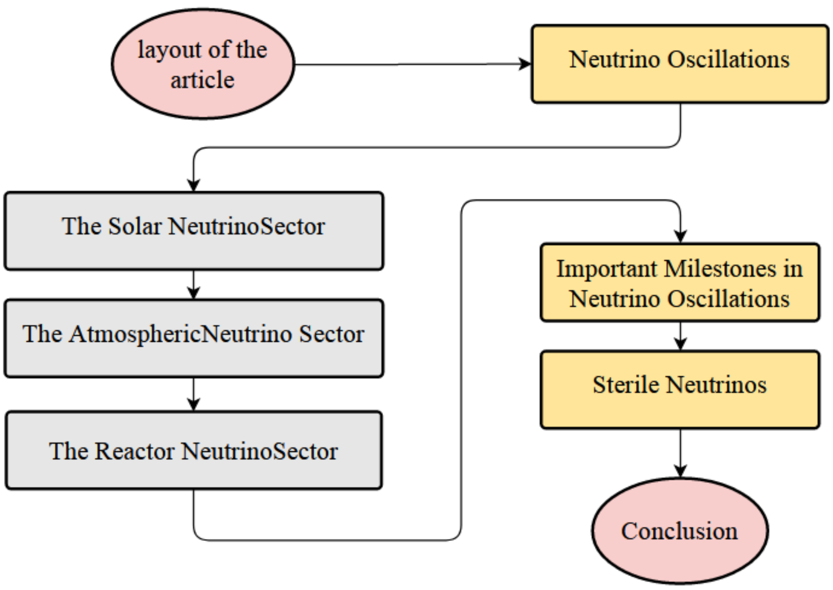


**Figure 1. Flowchart of the research methodology**

## 3- Neutrino Oscillations

In the Standard Model (SM) neutrinos are electrically neutral and massless. They only interact with other particles via weak interactions, which are described by the Lagrangian [10, 29]:

$$\mathcal{L}_{int} = -\frac{g}{4cos\theta_W}[\overline{\mathrm{V}}_e\gamma^\mu(1+\gamma_5)v_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e]Z_\mu \\ -\frac{g}{2\sqrt{2}}\overline{\mathrm{V}}_e\gamma^\mu(1+\gamma_5)eW_\mu^+ - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)v_eW_\mu^- \tag{11}$$

Charged current interaction Lagrangian contribution to neutrino potential in matter [10]:

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}j_p^{CC}W^p + h.c. \tag{12}$$

Neutral Current interaction Lagrangian contribution to neutrino potential in matter:

$$\mathcal{L}_{NC} = -\frac{g}{2cos\theta_W}j_p^{NC}Z^p \tag{13}$$

In the above relation, g is the $SU(2)_L$ gauge coupling constant, $\theta_W$ is the weak angle, and the charged and neutral currents $j_p^{CC}$ and $j_p^{NC}$ are given by;

$$j_p^{CC} = 2\sum_{l=\epsilon,\mu,\tau}V_{lL}Y_pl_L + \ldots \tag{14}$$

$$j_p^{NC} = \sum_{l=\epsilon,\mu,\tau}V_{lL}Y_pV_{lL} + \ldots \tag{15}$$

where *l* are the charged lepton fields and we have written only the terms containing the neutrino fields $v_l$. If neutrinos have non-zero masses, the left handed components $V_{\alpha L}$ of the neutrinos with the flavor $\alpha$ can also be expressed as a superposition if left handed states $V_{lL}$ of the neutrinos with masses given by $m_i$. Assuming that neutrinos are ultra-relativistic, we have

$$V_{\alpha L} = \sum_{i=1}^{N}U_{\alpha i V_{iL}} \tag{16}$$

where *U* is the unitary matrix defined earlier. If we consider a field operator which is used for creation and annihilation of particles and anti-particles, it implies that a flavor eigenstate $|v_\alpha|$ is a superposition of the different mass eigenstates $|v_i|$, according to [10, 18]:

$$|V_\alpha| = \sum_{i=}^{N}U_{\alpha i}^*\,|V_i| \tag{17}$$

### *3-1- The Solar Neutrino Sector:* $(sin^2\theta_{12}, \Delta m_{21}^2)$

The solar neutrino sector is the most important sector for all traditional solar neutrino experiments. The other important experiment is the reactor KamLAND experiment. These experiments are very sensitive in detecting the neutrino oscillations, which follows the dynamics of CPT-conservation. There are multiple experiments which detect and analyze data from solar neutrino. Some of these experiments are, Homestake [30], *Gallex/GNO* [31], *SAGE* [32]. These experiments are extremely sensitive to the rate of electron neutrinos which undergo interactions, but less sensitive to the energy or time of arrival of solar neutrinos at the detector. The neutrino experiment Kamiokande [33]. Results from Kamiokande also pointed in the direction that there is a solar neutrino deficit which was observed by the previous experiments. Its upgraded version Super-Kamiokande, which is about 10 times larger and more sensitive, has provided us with accurate results through observations in the last 20 years of operation. The interactions of the solar neutrinos with other particles and medium are detected through multiple scattering experiments but the elastic neutrino-electron scattering is studied often.

The elastic neutrino-electron scattering is sensitive to all neutrino flavors, in which the cross section for $v_\epsilon$ is large due to the effects of the charged-current neutrino-electron interaction. The correlation between the the recoil electron and the incident neutrino in the elastic scattering gives the informational details and the physics of the incoming neutrino in terms of its energy and the direction of arrival. Super-Kamiokande was very successful in its first three solar phases [34, 35]. Now, Super-Kamiokande is already in its fourth phase, where a very low energy detection threshold of 3.5 MeV has been achieved. In this period, Super-Kamiokande has already indicated that the effects of matter of the Earth has a big role in the physics of neutrino oscillations and solar neutrinos. The statistical result analyzed is of the order of $3\sigma$ [1, 2].

### *3-2- The Atmospheric Neutrino Sector:* $(sin^2\theta_{23}\Delta m^2_{31})$

The atmospheric neutrino is mostly generated in nucleon-decay experiments. The studies including atmospheric neutrino flux was measured in [1, 2]. In the earlier studies the experiments observed that there is a shortage of the number of atmospheric neutrinos than what was predicted in theory or models [36]. Later, the explanation came as an indication for the evidence of neutrino oscillations. This was seen in the observation of the zenith angle dependence for the muon-like neutrinos at Super-Kamiokande. Another aspect which was detected at the Super-Kamiokande was the dependence of the $L/E$ distribution of the data which is a characteristic of neutrino oscillations [37]. Currently, Super-Kamiokande is in its fourth stage of advancement. This experiment is capable of measuring atmospheric neutrino flux in the 100MeV to 1TeV range. At the Super-Kamiokande the neutrino events which are observed are basically as the following types:

- Fully Contained;
- Partially Contained;
- Upward going.

These three classifications depend on the topology of the detected event.

### *3-3- The Reactor Neutrino Sector*

Since neutrinos are relatively hard to detect, we can use reactors to find them. These reactors can produce enormous amounts of reactor neutrinos since they come in only one flavor called electron antineutrinos. These experiments can position their detectors at a range of many distances. This variety gives scientists plenty of time to observe and record how neutrinos can change properties. During this process, neutrinos undergo negative beta decay which happens inside the nucleus. This is when an unstable fission fragments along with an abundant amount of neutrons convert into protons, electrons, and antineutrinos. These neutrino oscillations experiments have measured antineutrino distribution that is the sum of antineutrino spectral shapes of all beta decays. Also antineutrino detection via reactor could also help remotely monitor the reactor itself, as they can provide real-time information within the reactor without being present in them [9, 18].

## 4- Important Milestones in Neutrino Oscillations

The results obtained from experiments related to the neutrino oscillations have been very successful, of various types and from various sources. These sources are solar and atmospheric neutrinos, reactor anti-neutrinos, neutrinos and anti-neutrinos from accelerator beams. The neutrino experiments make use of a variety of techniques, some of which are [32]:

- Radiochemical Methods;
- Water And Heavy Water Cherenkov Detectors;
- Liquid Scintillators;
- Plastic Scintillators;
- Streamer Chamber Detectors;
- Time Projection Chamber Detectors;
- Nuclear Emulsion detectors.

The principle results from this review paper are displayed in Figures 2 and 3. The experiments can be differentiated and explained as disappearance of a certain flavor and appearance of a certain flavor events [31-33]:

- The first results show how to measure flux of neutrinos having the same flavor as that produced at the source
- The second look for neutrinos of various flavors with respect to those emitted by the source.

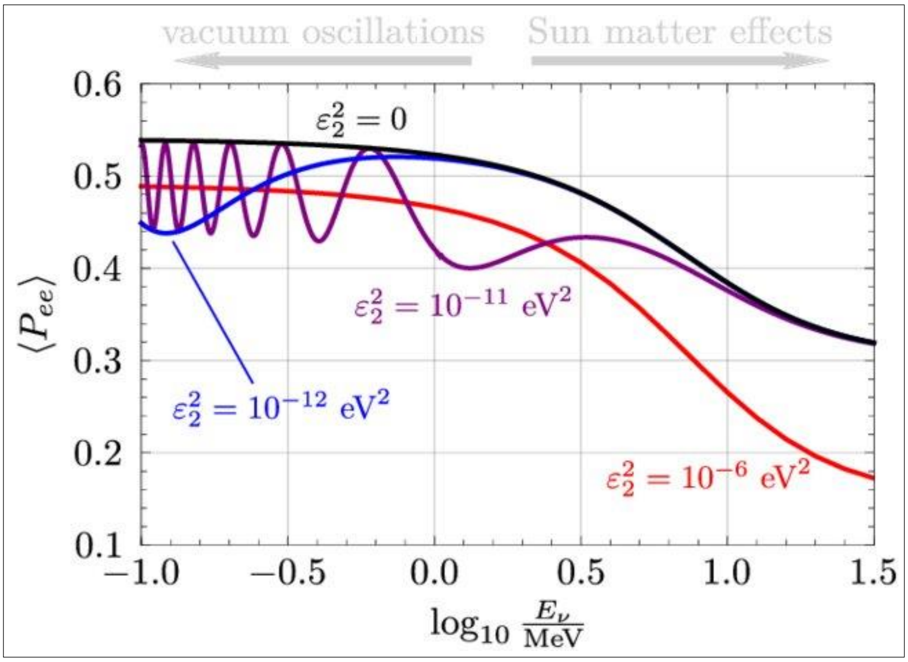


**Figure 2. Electron Neutrino survival probability at Earth detection for neutrinos produced in a solar plasma with matter potential [38]**

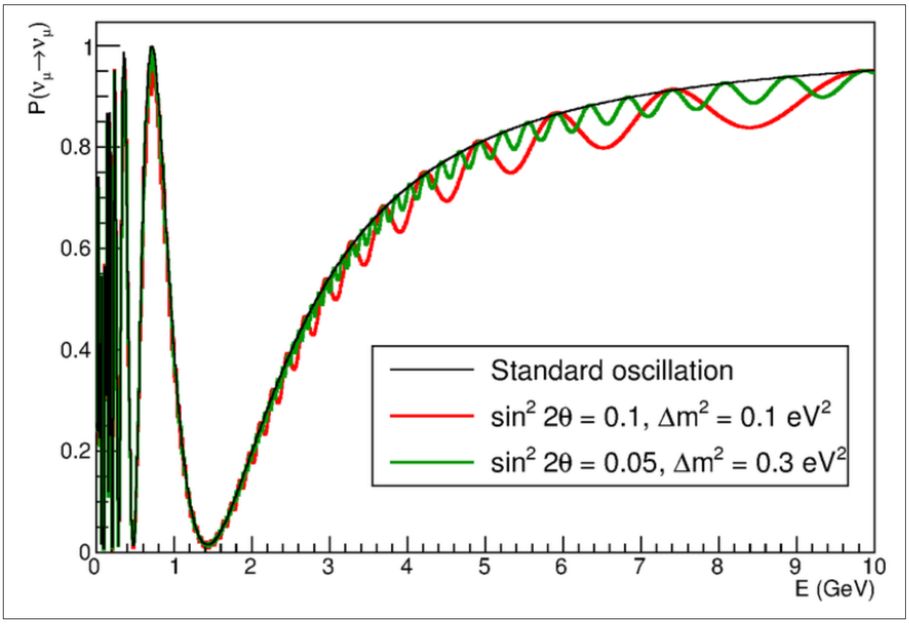


**Figure 3. Muon neutrino survival probability at a distance of 735km as a function of the neutrino energy, for different values of the sterile neutrino mass splitting and oscillation angle [39]**

The experiments studying the atmospheric neutrinos at Kamiokande, Super-Kamiokande and the solar neutrinos at SNO, all give superb evidence and provide excellent studies for neutrino oscillations. From Super-Kamiokande results, the atmospheric neutrino measurements have been extremely positive.

## 5- Sterile Neutrinos

There is a possibility that more than three light neutrino species participate in neutrino oscillations [40]. A deficit of atmospheric muon neutrinos was first observed by the Kamiokande experiment [35, 41] and IMB [42, 43] experiments. It has been widely discussed that this could be because of the possibility of oscillations of muon neutrinos into sterile neutrinos. The oscillations of active neutrinos into sterile neutrinos have also been widely studied as a possible explanation of the solar neutrino problem [46-49]. Even though there have been several explanations, there are still many neutrino oscillation anomalies which cannot be explained in the standard three flavor framework. The understanding of this anomaly requires more experiments, which have a higher sensitivity to detect sterile neutrinos or their interactions [50].

Among the many experimental setups which aim to look for evidences for sterile neutrinos, one such experiment is the Liquid Scintillator Neutrino Detector (LSND) [51]. The Liquid Scintillator Neutrino Detector (LSND) was designed to detect neutrinos originating in a proton tar- get and beam stop at the Los Alamos Meson Physics Facility (LAMPF). The primary goal was to search for transitions from muon-type to electron-type neutrinos in two complementary ways. The experimental setup is shown in Figure 4. Sterile neutrinos came into theory when new studies started to look deeply into the formulation and detection of dark matter. Sterile neutrinos completely differ from the neutrinos in the Standard Model. They do not interact with regular matter as neutrinos do and hence it makes it very difficult to detect or even look for any channel through which it interacts. Lawrence Livermore National Laboratory (LLNL) and Colorado School of Mines have been working towards the study of sterile neutrinos.

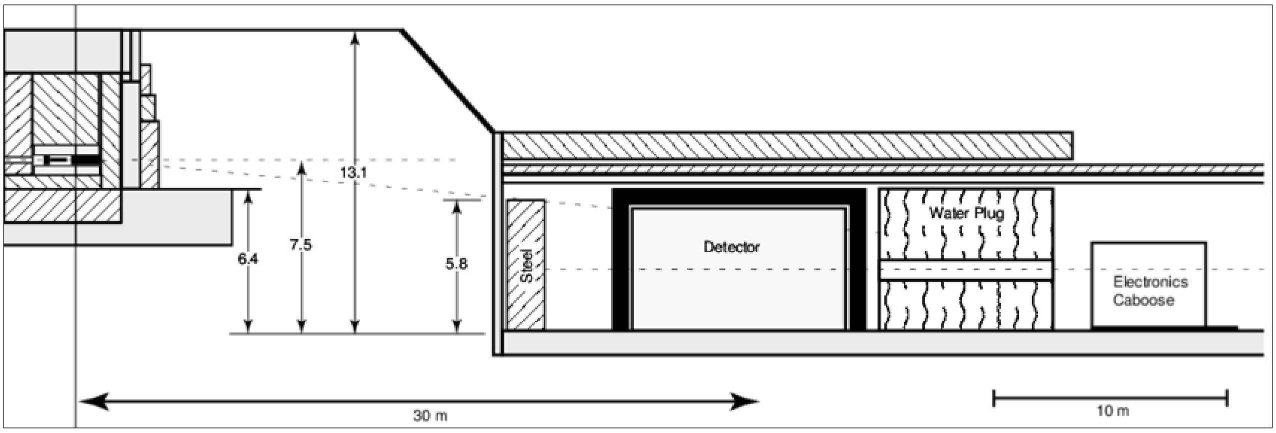


**Figure 4. Detector enclosure and target area configuration, elevation view**

### *5-1- Fundamentals of Sterile Neutrino Phenomenology*

The phenomenology and the formalism of the neutrinos is the Standard Model is similar if we talk about 3-flavors of neutrinos or extend it to more flavors. There are other studies related to this phenomenology [52, 53]. The weak interaction Lagrangian in the flavor basis [40] can be written as,

$$neu\mathcal{L} = \sum_{\alpha=e,\mu,\tau}\left[\frac{g}{\sqrt{2}}\left(\overline{\nu_\alpha, L}\gamma^\mu \nu_{\alpha,L} W_\mu^+ + h.c.\right) + \frac{g}{\sqrt{2\cos(\theta_\omega)}}\left(\overline{\nu_\alpha, L}\gamma^\mu \nu_{\alpha,L} Z_\mu\right)\right] \quad (18)$$

In the mass basis we can write the neutrino flavors as,

$$\nu_\alpha = \sum_{i=1}^{n+3} U_{\alpha i}\, \nu_i \quad (19)$$

where the summation goes to $n+3$ states considering there bare more than 3 neutrino flavors. $U_{\alpha i}$ is known as the mixing matrix which gives the transformation from the flavor eigenstates to the mass eigenstates. Quantum field theory, just like quantum mechanics makes use of creation operators. This creation operator is also applied in neutrino states. When a neutrino creation operator is applied on vacuum, it gives rise to a neutrino flavor state which is also known as the initial state of the neutrino flavor. This flavor state can also be connected to the mass eigenstates using the neutrino mixing matrix and is given by,

$$|\nu_\alpha\rangle = \sum_{i=1}^{n+3} U_{\alpha i}^*\, |\nu_i\rangle \quad (20)$$

where $U^*$ is the complex conjugate of U. Using the time evolution phenomenon, we can write Equation 19 as,

$$|\nu_\alpha(t,x)\rangle = \sum_{i=1}^{n+3} \exp\left(-iE_i t + ip_i x\right) U_{\alpha i}^* |\nu_i\rangle \quad (21)$$

where $E_i$ and $p_i$ are the energy and momentum of the i-th mass eigenstate. Since we know that neutrinos oscillate between flavors while travelling long distances. These oscillations occur regularly and have a certain probability which can be given as,

$$P_{\alpha\beta} \equiv \langle \nu_\alpha | \nu_\beta(t,x)\rangle = \sum_{i,j} U_{\alpha i}^*\, U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp(-i[E_i - E_j]t)\exp(i[p_i - p_j]x) \quad (22)$$

where the indices $i, j$ go from 1 to 3+n. There are multiple experiments which are currently working towards searching for the elusive sterile neutrino and some of those experiments have been mentioned in the previous sections. These various experiments apply a variety of techniques to detect sterile neutrinos. Some aspects of sterile neutrino detection are appearance searches and disappearance searches. These techniques are out of scope of this review.

## 6- Conclusion

The calculated probability of an electron neutrino and muon neutrino's survival was calculated based on the best-fit parameters for the electron and muon neutrinos. The probability of survival varies based on the time of day and also the distance of the neutrinos. The mass eigenvalues and the splitting angle values also play a pivotal role in those calculations. Currently, there are still many open problems that theorists and experiments are trying to solve. Some of them deal with sterile neutrinos, dark matter, and the connection of dark matter with neutrino oscillations. There are various experiments that are trying to understand the interactions related to neutrino oscillations, sterile neutrinos, and dark matter. There are also studies underway to understand if neutrinos are Dirac particles or Majorana particles.

The neutrino oscillations that are discussed here are connected to the probability of the oscillation of a certain flavor to another flavor. These results and predictions are for the three flavor oscillations in vacuum and in matter. Currently, neutrino oscillations are the subject of active research because these oscillations can give us an insight into understanding the Universe in its fundamental form. This can also lead us towards understanding dark matter and how dark matter shapes the universe at a larger scale. The experiments which are working towards understanding and detecting neutrinos are also looking for interactions which can give a deeper understanding of how neutrino properties can be used in strengthening the Standard Model of Particle Physics and look for scenarios beyond the Standard Model.

The future work is related to focusing on the current phenomenology of neutrinos and their connection to dark matter. There are various scenarios which have promising aspects of dark matter detection and multiple ways of understanding neutrino oscillations and dark matter. These aspects of neutrinos are part of our future work.

## 7- Declarations

### 7-1- Author Contributions

Conceptualization, K.J-J., A.R., C.B., and P.S.; writing—original draft preparation, K.J-J., A.R., and C.B.; writing—review and editing, P.S.; supervision, P.S. All authors have read and agreed to the published version of the manuscript.

### 7-2- Data Availability Statement

Data sharing is not applicable to this article.

### 7-3- Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

### 7-4- Acknowledgements

We thank Dr. Jacquline Dunn at Midwestern State University for her continuous support and providing the avenue for us to continue our research group. The authors, Kendra Jean Jacques and Anna Roland thank the physics program for the scholarships and opportunity to work on this research project.

### 7-5- Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

## 8- References

[1] Fukuda, Y., Hayakawa, T., Ichihara, E., Inoue, K., Ishihara, K., Ishino, H., ... Wilkes, R. J., Young, K. K. (1998). Super-Kamiokande Collaboration: Evidence for oscillation of atmospheric neutrinos. Physical Review Letters, 81(8), 1562-1567. doi:10.1103/PhysRevLett.81.1562.

[2] Ashie, Y., Hosaka, J., Ishihara, K., Itow, Y., Kameda, J., Koshio, Y., ... Washburn, K., and Wilkes, R. J. (2004). Super-Kamiokande Collaboration: Evidence for an oscillatory signature in atmospheric neutrino oscillations. Physical review letters, 93(10), 101801. doi:10.1103/PhysRevLett.93.101801.

[3] Pontecorvo, B. (1957). Mesonium and antimesonium. Soviet Journal of Experimental and Theoretical Physics, 6, 429-431.

[4] Pontecorvo, B. (1958). Inverse Beta Processes and Nonconservation of Lepton Charge. Joint Inst. of Nuclear Research, Zhur. Eksptl'. i Teoret. Fiz, 34. (In Russian).

[5] Fukuda, Y., Hayakawa, T., Ichihara, E., Inoue, K., Ishihara, K., Ishino, H., … Wilkes, R. J., Young, K. K. (1999). Super-Kamiokande Collaboration: Measurement of the flux and zenith-angle distribution of upward throughgoing muons by Super-Kamiokande. Physical Review Letters, 82(13), 2644. doi:10.1103/PhysRevLett.82.2644.

[6] Ahmad, Q. R., Allen, R. C., Andersen, T. C., Anglin, J. D., Barton, J. C., Beier, E. W., ... & Smith, A. R. (2002). Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory. Physical review letters, 89(1), 011301. doi:10.1103/PhysRevLett.89.011301.

[7] Eguchi, K., Enomoto, S., Furuno, K., Goldman, J., Hanada, H., Ikeda, H., ... Wang, Y.-F. (2003). KamLAND Collaboration: First results from KamLAND: evidence for reactor antineutrino disappearance. Physical Review Letters, 90(2), 021802. doi:10.1103/PhysRevLett.90.021802.

[8] Kaether, F., Hampel, W., Heusser, G., Kiko, J., & Kirsten, T. (2010). Reanalysis of the Gallex solar neutrino flux and source experiments. Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics, 685(1), 47–54. doi:10.1016/j.physletb.2010.01.030.

[9] Hayes, A. C., & Vogel, P. (2016). Reactor Neutrino Spectra: Annual Review of Nuclear and Particle Science, 66, 219–244. doi:10.1146/annurev-nucl-102115-044826.

[10] Sharma, P. (2014). Probing New Physics through Third Generation Leptons, Ph.D. Thesis, and University of Mississippi, Mississippi, United States.

[11] Abratenko, P., Alrashed, M., An, R., Anthony, J., Asaadi, J., Ashkenazi, A., ... & Spentzouris, P. (2020). Search for heavy neutral leptons decaying into muon-pion pairs in the MicroBooNE detector. Physical Review D, 101(5), 052001. doi:/doi.org/10.1103/PhysRevD.101.052001.

[12] Abe, Y., Aberle, C., Akiri, T., Dos Anjos, J. C., Ardellier, F., Barbosa, A. F., ... & Schwetz, T. (2012). Indication of reactor ν¯e disappearance in the Double Chooz experiment. Physical Review Letters, 108(13), 131801. doi:10.1103/PhysRevLett.108.131801.

[13] Davis, R., Harmer, D. S., & Hoffman, K. C. (1968). Search for neutrinos from the sun. Physical Review Letters, 20(21), 1205–1209. doi:10.1103/PhysRevLett.20.1205.

[14] An, F. P., Bai, J. Z., Balantekin, A. B., Band, H. R., Beavis, D., Beriguete, W., ... & Morgan, J. E. (2012). Observation of electron-antineutrino disappearance at Daya Bay. Physical Review Letters, 108(17), 171803. doi:10.1103/PhysRevLett.108.171803.

[15] Aliu, E., Andringa, S., Aoki, S., Argyriades, J., Asakura, K., Ashie, R., ... & Yamada, S. (2005). The K2K Collaboration collaboration: Evidence for muon neutrino oscillation in an accelerator-based experiment. Physical review letters, 94(8), 081802.

[16] Michael, D. G., Adamson, P., Alexopoulos, T., Allison, W. W. M., Alner, G. J., Anderson, K., ... & McDonald, J. (2006). Observation of muon neutrino disappearance with the MINOS detectors in the NuMI neutrino beam. Physical review letters, 97(19), 191801. doi:10.1103/PhysRevLett.97.191801

[17] Cleveland, B. T., Daily, T., Davis Jr, R., Distel, J. R., Lande, K., Lee, C. K., ... & Ullman, J. (1998). Measurement of the solar electron neutrino flux with the Homestake chlorine detector. The Astrophysical Journal, 496(1), 505. doi:10.1086/305343.

[18] Rashed, A., Sharma, P., & Datta, A. (2013). Tau-neutrino as a probe of nonstandard interaction. Nuclear Physics B, 877(3), 662–682. doi:10.1016/j.nuclphysb.2013.10.022.

[19] Duraisamy, M., Sharma, P., & Datta, A. (2014). Azimuthal B →d∗τ- ν - τ angular distribution with tensor operators. Physical Review D - Particles, Fields, Gravitation and Cosmology, 90(7), 74013. doi:10.1103/PhysRevD.90.074013.

[20] Abdurashitov, J. N., Gavrin, V. N., Gorbachev, V. V., Gurkina, P. P., Ibragimova, T. V., Kalikhov, A. V., ... Wilkerson, J. F. . (2009). SAGE Collaboration: Measurement of the solar neutrino capture rate with gallium metal. III. Results for the 2002–2007 data-taking period. Physical Review C, 80(1), 015807. doi:0.1103/PhysRevC.80.015807.

[21] Auger, M., Berner, R., Chen, Y., Ereditato, A., Goeldi, D., Koller, P. P., ... & Asaadi, J. (2019). A New Concept for Kilotonne Scale Liquid Argon Time Projection Chambers. arXiv preprint arXiv:1908.10956.

[22] Abratenko, P., An, R., Anthony, J., Arellano, L., Asaadi, J., Ashkenazi, A., ... & Snider, E. L. (2021). First Measurement of Energy-dependent Inclusive Muon Neutrino Charged-Current Cross Sections on Argon with the MicroBooNE Detector. arXiv preprint arXiv:2110.14023.

[23] MicroBooNE Collaboration. (2016). Selection and kinematic properties of νμ charged-current inclusive events in 5 × 1019 POT of MicroBooNE data. Available online: http://microboone.fnal.gov/wp-content/uploads/MICROBOONE-NOTE-1010-PUB.pdf (accessed on December 2021).

[24] Ydrefors, E., & Suhonen, J. (2012). Charged-current neutrino-nucleus scattering off the even molybdenum isotopes. In Advances in High Energy Physics (Vol. 2012), 1-12. doi:10.1155/2012/373946.

[25] Adams, C., Alrashed, M., An, R., Anthony, J., Asaadi, J., Ashkenazi, A., ... & Tang, W. (2018). Rejecting cosmic background for exclusive neutrino interaction studies with Liquid Argon TPCs; a case study with the MicroBooNE detector. arXiv preprint arXiv:1812.05679.

[26] Adams, C., Alrashed, M., An, R., Anthony, J., Asaadi, J., Ashkenazi, A., ... & Tagg, N. (2019). First measurement of ν μ charged-current π 0 production on argon with the MicroBooNE detector. Physical Review D, 99(9), 091102. doi:10.1103/PhysRevD.99.091102.

[27] The MicroBoone Collaboration. (2018). Automated selection of electron neutrinos from the numi beam in the microboone detector and prospects for a measurement of the charged-current inclusive cross section. Available online: https://microboone.fnal.gov/wp-content/uploads/MICROBOONE-NOTE-1054-PUB.pdf (accessed on December 2021).

[28] Abratenko, P., Alrashed, M., An, R., Anthony, J., Asaadi, J., Ashkenazi, A., ... & Soderberg, M. (2020). First Measurement of Differential Charged Current Quasielasticlike ν μ-Argon Scattering Cross Sections with the MicroBooNE Detector. Physical review letters, 125(20), 201803. doi:10.1103/PhysRevLett.125.201803.

[29] Hirata, K. S., Inoue, K., Ishida, T., Kajita, T., Kihara, K., Nakahata, M., Nakamura, K., Ohara, S., Sato, N., Suzuki, Y., Totsuka, Y., Yaginuma, Y., Mori, M., Oyama, Y., Suzuki, A., Takahashi, K., Yamada, M., Koshiba, M., Nishijima, K., … Zhang, W. (1991). Real-time, directional measurement of B8 solar neutrinos in the Kamiokande II detector. Physical Review D, 44(8), 2241–2260. doi:10.1103/PhysRevD.44.2241.

[30] Hosaka, J., Ishihara, K., Kameda, J., Koshio, Y., Minamino, A., Mitsuda, C., ... &. Wilkes, R. J. (2006). Super-Kamiokande Collaboration: Solar neutrino measurements in Super-Kamiokande-I. Physical Review D, 73(11), 112001. doi:10.1103/PhysRevD.73.112001.

[31] Cravens, J.P., Abe, K., Iida, T., Ishihara, K., Kameda, J., Koshio, Y.,... &. Wilkes, R.J. (2008). Super-Kamiokande Collaboration: Solar neutrino measurements in Super-Kamiokande-II. Physical review D, 78(3), 032002. doi:10.1103/PhysRevD.78.032002.

[32] Abe, K., Hayato, Y., Iida, T., Ikeda, M., Ishihara, C., Iyogi, K., ... &. Wilkes, R.J. (2011). Super-Kamiokande Collaboration. Solar neutrino results in Super-Kamiokande-III. Physical Review D, 83(5), 052010. doi:10.1103/PhysRevD.83.052010.

[33] Abe, K., Haga, Y., Hayato, Y., Ikeda, M., Iyogi, K., Kameda, J., ... & Calland, R. G. (2016). Solar neutrino measurements in Super-Kamiokande-IV. Physical Review D, 94(5), 052010. doi:10.1103/PhysRevD.94.052010.

[34] Davies, A. T., Froggatt, C. D., & Moorhouse, R. G. (1996). Electroweak baryogenesis in the next to minimal supersymmetric model. Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics, 372(1–2), 88–94. doi:10.1016/0370-2693(96)00076-7.

[35] Hirata, K. S., Inoue, K., Ishida, T., Kajita, T., Kihara, K., Nakahata, M., ... & Zhang, W. (1992). Observation of a small atmospheric νμ/νe ratio in Kamiokande. Physics Letters B, 280(1-2), 146-152. doi:10.1016/0370-2693(92)90788-6.

[36] de Salas, P. F., Forero, D. V., Ternes, C. A., Tórtola, M., & Valle, J. W. F. (2018). Status of neutrino oscillations 2018: 3σ hint for normal mass ordering and improved CP sensitivity. Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics, 782, 633–640. doi:10.1016/j.physletb.2018.06.019.

[37] Wallraff, M., & Wiebusch, C. (2015). Calculation of oscillation probabilities of atmospheric neutrinos using nuCraft. Computer Physics Communications, 197, 185–189. doi:10.1016/j.cpc.2015.07.010.

[38] Anamiati, G., Fonseca, R. M., & Hirsch, M. (2018). Quasi-Dirac neutrino oscillations. Physical Review D, 97(9), 095008. doi:10.1103/PhysRevD.97.095008.

[39] Bridle, S., Elvin-Poole, J., Evans, J., Fernandez, S., Guzowski, P., & Söldner-Rembold, S. (2017). A combined view of sterile-neutrino constraints from CMB and neutrino oscillation measurements. Physics Letters B, 764, 322-327. doi:10.1016/j.physletb.2016.11.050.

[40] Dasgupta, B., & Kopp, J. (2021). Sterile neutrinos. Physics Reports, 928, 1–63. doi:10.1016/j.physrep.2021.06.002.

[41] Fukuda, Y., Hayakawa, T., Inoue, K., Ishida, T., Joukou, S., Kajita, T., ... & Zhang, W. (1994). Atmospheric νμνe ratio in the multi-GeV energy range. Physics Letters B, 335(2), 237-245. doi:10.1016/0370-2693(94)91420-6.

[42] Casper, D., Becker-Szendy, R., Bratton, C. B., Cady, D. R., Claus, R., Dye, S. T., Gajewski, W., Goldhaber, M., Haines, T. J., Halverson, P. G., Jones, T. W., Kielczewska, D., Kropp, W. R., Learned, J. G., Losecco, J. M., McGrew, C., Matsuno, S., Matthews, J., Mudan, M. S., … Van Der Velde, J. C. (1991). Measurement of atmospheric neutrino composition with the IMB-3 detector. Physical Review Letters, 66(20), 2561–2564. doi:10.1103/PhysRevLett.66.2561.

[43] Kajita, T., Kearns, E., & Shiozawa, M. (2016). Establishing atmospheric neutrino oscillations with Super-Kamiokande. Nuclear Physics B, 908, 14–29. doi:10.1016/j.nuclphysb.2016.04.017.

[44] Akhmedov, E., Lipari, P., & Lusignoli, M. (1993). Matter effects in atmospheric neutrino oscillations. Physics Letters B, 300(1–2), 128–136. doi:10.1016/0370-2693(93)90759-B.

[45] Liu, Q. Y., & Smirnov, A. Y. (1998). Neutrino mass spectrum with $\nu\mu \rightarrow \nu s$ oscillations of atmospheric neutrinos. Nuclear Physics B, 524(3), 505–523. doi:10.1016/S0550-3213(98)00269-7.

[46] Peltoniemi, J. T., & Valle, J. W. F. (1993). Reconciling dark matter, solar and atmospheric neutrinos. Nuclear Physics, Section B, 406(1–2), 409–422. doi:10.1016/0550-3213(93)90174-N.

[47] Fuller, G. M., Primack, J. R., & Qian, Y. Z. (1995). Do experiments and astrophysical considerations suggest an inverted neutrino mass hierarchy? Physical Review D, 52(2), 1288–1291. doi:10.1103/PhysRevD.52.1288.

[48] Gomez-Cadenas, J. J., & Gonzalez-Garcia, M. C. (1996). Futurev $\tau$ oscillation experiments and present data. Zeitschrift für Physik C Particles and Fields, 71(3), 443-454. doi:10.1007/BF02907002.

[49] Okada, N., & Yasuda, O. (1997). A sterile neutrino scenario constrained by experiments and cosmology. International Journal of Modern Physics A, 12(21), 3669–3694. doi:10.1142/S0217751X97001894.

[50] Dasgupta, B., & Kopp, J. (2021). Sterile neutrinos. Physics Reports, 928, 1–63. doi:10.1016/j.physrep.2021.06.002.

[51] Athanassopoulos, C., Auerbach, L. B., Bauer, D., Bolton, R. D., Burman, R. L., Cohen, I., ... & Yellin, S. (1997). The Liquid scintillator neutrino detector and LAMPF neutrino source. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 388(1-2), 149-172. doi:10.1016/S0168-9002(96)01155-2.

[52] Akhmedov, E. K. (2000). Neutrino Physics. arXiv preprint hep-ph/0001264. Available online: https://arxiv.org/pdf/hep-ph/0001264.pdf (accessed on January 2022).

[53] Beuthe, M. (2003). Oscillations of neutrinos and mesons in quantum field theory. Physics Report, 375(2–3), 105–218. doi:10.1016/S0370-1573(02)00538-0.