

Control of Constrained Linear-Time Varying Systems via Kautz Parametrization of Model Predictive Control Scheme

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Abstract

Kautz parametrization of the Model Predictive Control (MPC) method has shown its ability to reduce the number of decision variables in Linear Time Invariant (LTI) systems. This paper devotes to extend Kautz network to be used in MPC Algorithm for linear time-varying systems. It is shown that Kautz network enables us to maintain a satisfactory performance while the number of decision variables are reduced considerably. Stability of the algorithm is studied under the framework of the optimal solution. The proposed method is validated by an illustrative example. In this regard, the performance of unconstrained systems as well as constrained ones is compared.

Keywords:

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Kautz Network;
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1- Introduction

Keeping computation times, consequently the sampling time, in Model Predictive Control (MPC) as small as possible is a continuous challenge. In order to address this issue, different approaches have been proposed by numerous authors. Explicit MPC, for instance, replaces the on-line numerical solution of an optimization problem of an LTI system with a look-up table [1, 2], with precomputed coefficients. Wang [3] reduced the number of decision variables in the continuous-time domain by exploiting Laguerre functions in the problem. Later she extended this method to discrete-time systems and showed its effectiveness in reducing the number of decision variables as well as in dealing with constraints on inputs and states [4]. In a different approach the idea of using Kautz network was introduced in the context of MPC for LTI systems [5]. Then, numerous papers, all for LTI systems, have been published based on those ideas. It has been shown that the Kautz parametrization increases the volume of the feasible region [6]. In the industrial application, authors of [7] used the method in explicit MPC to design and implement an MPC method on PLCs. The method has shown its effectiveness in robust min-max problem of MPC scheme [8].

In all of the abovementioned works, it was considered that the system is LTI and the goal was to reduce the computational time problem in MPC via reducing the total number of decision variables. However, for Linear Time-Varying (LTV) systems the transient nature of the systems does not permit us to use fixed methods, such as explicit MPC, to reduce the computation times. For LTV systems the general solution of the Linear Quadratic Regulator (LQR) problem demands solving the Ricatti equation backward over an infinity horizon that is computationally challenging because the solution of the Ricatti equation for LTV systems does not converge to a fixed value, contrary to LTI systems. To address the computational difficulties of LTV systems, [9] solved the optimal problem over a finite time interval and used the receding horizon principle in the continuous-time domain. This method was used in [10] for the discrete-time domain. A more computationally efficient method was presented in [11] to generate the control law for [9] and [10] in an alternative manner. These works used the receding horizon principle to solve the on-line optimization problem of

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LTV systems over a finite time interval without referring directly to MPC. The addressed method was employed for an unconstrained LTV system. The authors demonstrated the stability of the scheme by enforcing the final constraint to be the origin. Decrease in the computation times for LTV systems was the focal motivation there; therefore, the authors of [9-11] were not interested in imposing constraints on the states and inputs. The only constraint used in these works (equality terminal constraint) was due to the need of ensuring stability. To the best of our knowledge, input and state constraints were introduced by [12] into the receding horizon optimal control problem of a time-varying system. In the presence of input and state constraints, the authors established the stability of the system by selecting the length of the horizon long enough to resemble infinite horizon and applying a terminal equality constraint. A similar, yet independent, approach for continuous-time nonlinear systems was adopted by [13] where the stability was proved using a sole terminal equality constraint (no terminal penalty function was employed). To reduce the conservatism of the method for general time-varying systems, the authors of [14] employed a terminal state penalty as well as a terminal inequality constraint. In an implementation of the method, [15] obtained an LTV system via successive linearization of the nonlinear vehicle equations of a vehicle around a nominal maneuvering trajectory, then applied the MPC method of time varying systems on the linearized system.

This paper is devoted to the reduction of computational demands of model predictive control for linear time-varying systems. It is shown that the extension of Kautz parametrization from LTI to LTV systems allows us to reduce the total number of unknown variables in the optimizer vector that leads to a decrease in the computational load. The time-varying nature of the system does not permit us to employ the augmentation method of [4] for the control input; therefore, a different method for augmentation is used here. The efficiency of the proposed method to decrease the number of decision variables without impairing the system performance is illustrated via a simulation example. The stability of the proposed method is established via a terminal equality constraint.

The rest of the paper is organized as follows: In Section 2, model predictive control of LTV systems is presented. Section 3 briefly introduces the Kautz network and then describes how to parametrize the input of the model predictive control problem for LTV systems to be consistent with the Kautz network. A discussion on the stability is provided in Section 4. Section 5 illustrates the application of the method and its advantages by providing an example, followed by conclusions in Section 6.

2- MPC Method for LTV Systems

Consider the linear time-varying system

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ x(0) &= x_0 \end{aligned} \quad (1)$$

Where $x(\cdot) \in \mathbb{R}^n$ is the state vector, $u(\cdot) \in \mathbb{R}^m$ is the input vector, $A(\cdot) \in \mathbb{R}^{n \times n}$ is a time varying system matrix and $B(\cdot) \in \mathbb{R}^{n \times m}$ is a time varying input matrix. Under a feasible control input $u(\cdot)$, the solution of (1) is given by [16]:

$$x_0(k+1) = \Phi(k+1,0)x_0 + \sum_{\tau=0}^k \Phi(k+1,\tau+1) B(\tau)u(\tau) \quad (2)$$

Where $\Phi(\cdot, \cdot) \in \mathbb{R}^{n \times n}$ is called the state transition matrix and is defined as

$$\begin{aligned} \Phi(k, k_0) &= A(k-1)A(k-2) \dots A(k_0) \\ \Phi(k_0, k_0) &= I \end{aligned} \quad (3)$$

Employing (1)-(3), we are able to write the prediction vector X with prediction horizon N_p as an explicit function of the future control sequence vector U and the present state $x(0) = x_0$ as

$$X = S_X x_0 + S_U U \quad (4)$$

Where $X \in \mathbb{R}^{n(N_p+1)}$, $U \in \mathbb{R}^{mN_p}$, $S_X \in \mathbb{R}^{n(N_p+1) \times n}$ and $S_U \in \mathbb{R}^{n(N_p+1) \times mN_p}$ are given by

$$S_U = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ \Phi(1,1)B(0) & 0 & 0 & \cdots & \cdots & 0 \\ \Phi(2,1)B(0) & \Phi(2,2)B(1) & 0 & \cdots & \cdots & 0 \\ \Phi(3,1)B(0) & \Phi(3,2)B(1) & \Phi(3,3)B(2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \Phi(N_p,1)B(0) & \Phi(N_p,2)B(1) & \Phi(N_p,3)B(2) & \cdots & \cdots & \Phi(N_p,N_p)B(N_p-1) \end{bmatrix}$$

The receding horizon optimal controller of system (1) is obtained by solving the constrained optimization problem given by

$$\begin{aligned} J(x_0) &= \min_U \sum_{k=0}^{N_p-1} \|x(k)\|_{Q(k)} + \|u(k)\|_{R(k)} + \|x(N_p)\|_P \\ \text{Subject to } & x(k+1) = A(k)x(k) + B(k)u(k) \\ & \underline{u}(k) \leq u(k) \leq \bar{u}(k), \quad k = 0, \dots, N_p - 1 \\ & \underline{x}(k) \leq x(k) \leq \bar{x}(k), \quad k = 0, \dots, N_p, \end{aligned} \quad (5)$$

In which $\|x\|_Q = x^T Q x$, and all system matrices, weight functions and constraints are considered to be time varying. For all $k \in \{0, 1, \dots, N_p - 1\}$, the weight matrices $Q(\cdot)$, $R(\cdot)$ and P are symmetric and positive definite. The optimization problem (5) can be expressed using the notation of (4) as

$$\begin{aligned} J(x_0) &= \min_U X^T \bar{Q} X + U^T \bar{R} U \\ \text{Subject to } & X = S_X x_0 + S_U U \\ & \underline{U} \leq U \leq \bar{U}, \\ & \underline{X} \leq X \leq \bar{X}, \end{aligned} \quad (6)$$

Where $\bar{Q}, \bar{R}, \underline{U}, \bar{U}, \underline{X}, \bar{X}$ are defined as

$$\begin{aligned} \bar{Q} &= \text{blockdiag}\{Q(0), Q(1), \dots, Q(N_p - 1), P\} \\ \bar{R} &= \text{blockdiag}\{R(0), R(1), \dots, R(N_p - 1)\} \\ \bar{X}^T &= [\bar{x}(0)^T \quad \bar{x}(1)^T \quad \bar{x}(2)^T \quad \dots \quad \bar{x}(N_p)^T] \\ \underline{X}^T &= [\underline{x}(0)^T \quad \underline{x}(1)^T \quad \underline{x}(2)^T \quad \dots \quad \underline{x}(N_p)^T] \\ \bar{U}^T &= [\bar{u}(0)^T \quad \bar{u}(1)^T \quad \bar{u}(2)^T \quad \dots \quad \bar{u}(N_p - 1)^T] \\ \underline{U}^T &= [\underline{u}(0)^T \quad \underline{u}(1)^T \quad \underline{u}(2)^T \quad \dots \quad \underline{u}(N_p - 1)^T] \end{aligned} \quad (7)$$

The optimal unconstrained vector of future decision variables U_0 and optimal cost (6) can be easily found as

$$U_0 = -(S_U^T \bar{Q} S_U + \bar{R})^{-1} S_U^T \bar{Q} S_X x_0 \quad (8)$$

$$J(x_0) = x_0^T (S_X^T \bar{Q} S_X - S_X^T \bar{Q} S_U (S_U^T \bar{Q} S_U + \bar{R})^{-1} S_U^T \bar{Q} S_X) x_0 \quad (9)$$

In a similar manner, we can find the optimal decision vector and the optimal cost value by employing dynamic programming and solving the discrete-time Ricatti equation.

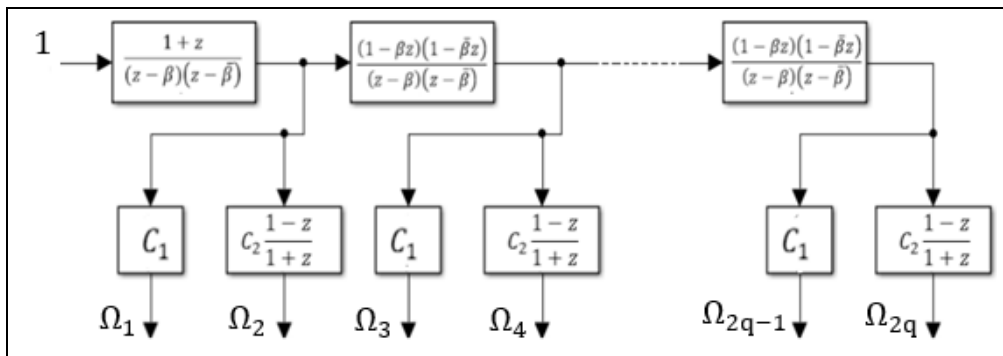


Figure 1. Illustration of Kautz network

$$P(k) = Q(k) + A(k)^T P(k+1)A(k) - A(k)^T P(k+1)B(k)[R(k) + B(k)^T P(k+1)B(k)]^{-1}B(k)^T P(k+1)A(k), \\ P(N_p) = P$$

Recursively for $k = N_p - 1, N_p - 2, \dots, 1, 0$. Then the corresponding control signal is:

$$u(k) = -[R(k) + B(k)^T P(k+1)B(k)]^{-1}B(k)^T P(k+1)A(k) x(k).$$

The main benefit of the dynamic programming approach and solving the Ricatti equation is that we can use them to show stability in a Lyapunov sense.

3- Kautz Function and Kautz Network in MPC-LTV

Belonging to the large family of orthonormal functions, the Kautz network was originally introduced as a means for system identification [17]. The Kautz transfer functions in the z -domain is [17]:

$$\Omega_{2q-1}(z) = |1 - \xi| \sqrt{\frac{1 - \xi\xi^*}{2}} \frac{1 + z}{(z - \xi)(z - \xi^*)} \left[\frac{(1 - \xi z)(1 - \xi^* z)}{(z - \xi)(z - \xi^*)} \right]^{q-1} \quad (10.a)$$

$$\Omega_{2q}(z) = |1 + \xi| \sqrt{\frac{1 - \xi\xi^*}{2}} \frac{1 - z}{(z - \xi)(z - \xi^*)} \left[\frac{(1 - \xi z)(1 - \xi^* z)}{(z - \xi)(z - \xi^*)} \right]^{q-1} \quad (10.b)$$

Where in q is the order of the functions. The parameters ξ, ξ^* are two complex-conjugate poles of the functions. In order to produce a stable transfer function, the poles of the network should be inside the unit circle $|\xi| < 1$. A set of Kautz functions $\{\Omega_q | q = 1, \dots, N\}$ is called a Kautz network of degree N , where N must be an even number, and it constitutes a set of orthonormal basis functions (see Figure 1). The time evolution of the state space representation of the Kautz network for an impulse input can be easily obtained from (10) and it is given by

$$L_1(k+2) = \Psi_1 L_1(k+1) + \Psi_2 L_1(k) \quad (11.a)$$

$$L_2(k+2) = \Psi_1 L_2(k+1) + \Psi_2 L_2(k) \quad (11.b)$$

$$\Psi_1 = \begin{bmatrix} p & 0 & 0 & \dots & 0 \\ p(r-1) & p & 0 & \dots & 0 \\ rp(r-1) & p(r-1) & p & \dots & 0 \\ r^2 p(r-1) & rp(r-1) & p(r-1) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{N-2} p(r-1) & r^{N-3} p(r-1) & \dots & p(r-1) & p \end{bmatrix}_{N \times N} \quad (11.c)$$

$$\Psi_2 = \begin{bmatrix} -r & 0 & 0 & \dots & 0 \\ 1 - r^2 & -r & 0 & \dots & 0 \\ r(1 - r^2) & 1 - r^2 & -r & \dots & 0 \\ r^2(1 - r^2) & r(1 - r^2) & 1 - r^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{N-2}(1 - r^2) & r^{N-3}(1 - r^2) & \dots & 1 - r^2 & -r \end{bmatrix}_{N \times N} \quad (11.d)$$

$$r = \xi\xi^* ; \quad p = \xi + \xi^* ; \quad C_1 = |1 - \xi| \sqrt{\frac{1 - r}{2}} ; \quad C_2 = |1 + \xi| \sqrt{\frac{1 - r}{2}}$$

Where $L_1(k) = [l_1(k) \quad l_3(k) \quad \dots \quad l_{2N-1}(k)]^T$ and $L_2(k) = [l_2(k) \quad l_4(k) \quad \dots \quad l_{2N}(k)]^T$ denote the odd and even Kautz functions. The initial conditions for the odd and even Kautz functions are given by:

$$L_1(0) = C_1 \begin{bmatrix} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{N-1} \end{bmatrix}, L_1(1) = C_1 \begin{bmatrix} 1+p \\ r+p(2r-1) \\ r[r+p(3r-2)] \\ r^2[r+p(4r-3)] \\ \vdots \\ r^{N-2}[r+p(Nr-N+1)] \end{bmatrix} \quad (11.e)$$

$$L_2(0) = -C_2 \begin{bmatrix} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{N-1} \end{bmatrix}, L_2(1) = C_2 \begin{bmatrix} 1-p \\ r+p(1-2r) \\ r[r+p(2-3r)] \\ r^2[r+p(3-4r)] \\ \vdots \\ r^{N-2}[r+p(N-1-Nr)] \end{bmatrix} \quad (11.f)$$

It should be mentioned that the absolute value of the Kautz pole $|\xi|$ adjusts the decaying rate as well as the oscillation of the Kautz functions. As it is illustrated in Figure 2, a pair of poles with a smaller absolute value indicates a faster convergence to zero while as the magnitude of the poles increases to the boundary of the unit disk, the values of $k_i(\cdot)$ need more steps to converge to zero. The location of the poles also indicates the number of steps that are needed for achieving the orthonormality of the Kautz network in the state space domain.

The author of [5] used Kautz networks in model predictive control for linear time invariant (LTI) systems. The major virtue, reported by the author, is to decrease the number of decision variables and consequently increase the speed of on-line optimization calculations. The main idea in Kautz parametrization is to consider the control signal as an impulse response of a stable system and then reconstruct it with a Kautz network [5]. Because of the parametrization by a Kautz network, the reconstructed control signal must decay to zero, so we have to convert the control input from u to Δu by incorporating an integrator into the system (1) and form the augmented system:

$$\begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} = \begin{bmatrix} A(k) & B(k) \\ 0 & I_{m \times m} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} B(k) \\ I_{m \times m} \end{bmatrix} \Delta u(k), \quad (12)$$

For simplicity, we use the notation

$$z(k+1) = A_z(k)z(k) + B_z(k)\Delta u(k), \quad (13)$$

where $z(k+1)^T = [x(k+1)^T \ u(k)^T]$ and $A_z(k), B_z(k)$ are the corresponding matrices to those given in (12). We will use this model instead of (1) to build the prediction vector (4) and to formulate the optimization problem (5)-(7).

The augmentation (12) makes the system inputs suitable for Kautz network. By this means, the total number of decision variables reduces from mN_p to $\sum_{j=1}^m N_j$ (where N_j is the degree of the Kautz network used to capture the control signal of channel j , $j = 1, \dots, m$). For every single input $\Delta u^{(j)}$, $j = 1, \dots, m$, of (13) we can write:

$$\Delta u^{(j)}(k) = \sum_{q=1}^{N_j} l_q^{(j)}(k) \eta_q^{(j)} \quad (14)$$

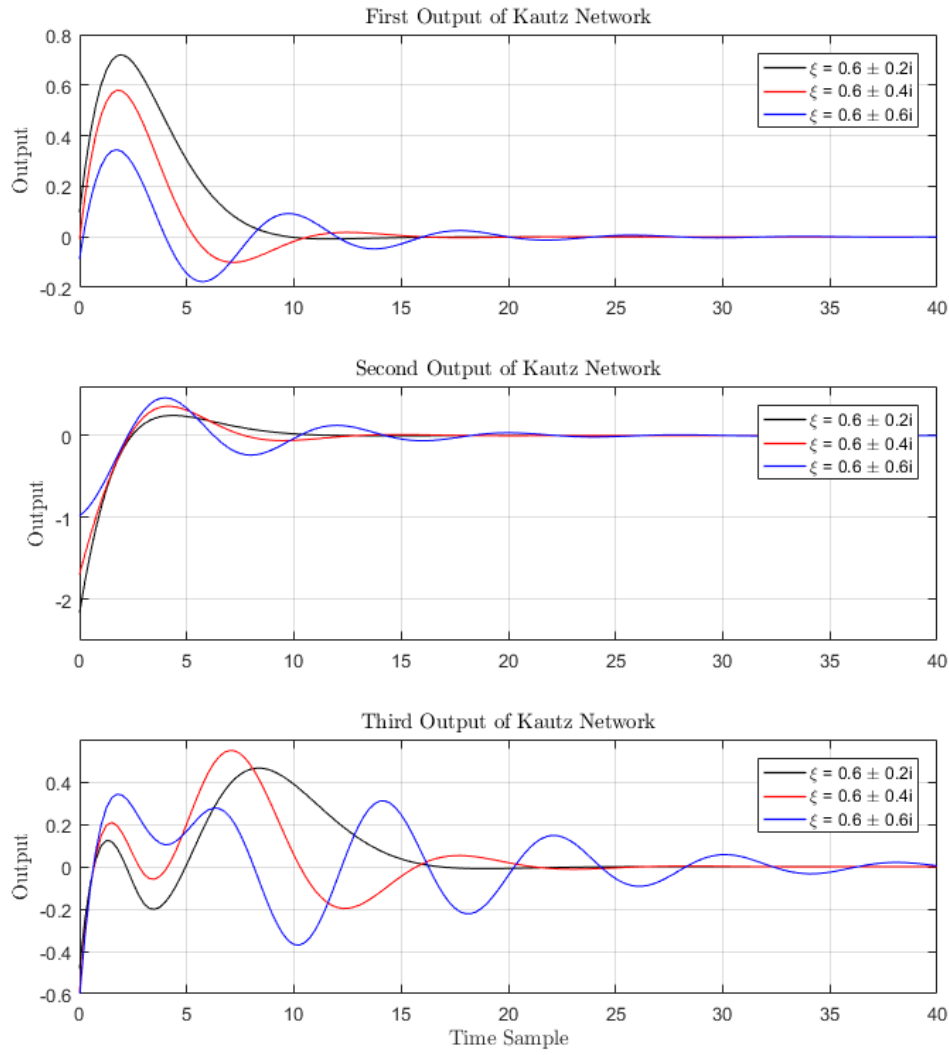


Figure 2. Output of Kautz network in time domain for different pairs of poles

By substituting (14) in (13), we can establish an optimization problem similar to (5)–(7) which is given by

$$\begin{aligned} J(x_0) &= \min_{\eta} X^T \bar{Q} X + \eta^T \Gamma^T \bar{R} \Gamma \eta \\ \text{Subject to } X &= S_X x_0 + S_U \Gamma \eta \\ G \eta &\leq g \end{aligned} \quad (15)$$

Where X, \bar{Q}, \bar{R} are defined in (7) and G, g are the corresponding time-varying constraint matrix and constraint vector of the optimization problem. For a system with m inputs, the vector η and the matrix Γ are defined as

$$\begin{aligned} \eta^T &= [\eta^{(1)T} \quad \eta^{(2)T} \quad \dots \quad \eta^{(m)T}], \\ \Gamma^T &= [L(0)^T \quad L(1)^T \quad \dots \quad L(N_p - 1)^T], \\ L(k) &= \begin{bmatrix} L^{(1)}(k) & 0 & \dots & 0 \\ 0 & L^{(2)}(k) & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & \dots & L^{(m)}(k) \end{bmatrix}. \end{aligned} \quad (16)$$

In (16), $L^{(p)}(k)$ is the k 'th output of the Kautz network for the p 'th input ($p = 1, \dots, m$) and is given by the left hand side of (11). Solving the optimization problem (15), the unconstrained optimal decision variables η_0 and their corresponding ΔU_0 are given by:

$$\eta_0 = -(\Gamma^T S_U^T \bar{Q} S_U \Gamma + \Gamma^T \bar{R} \Gamma)^{-1} \Gamma^T S_U^T \bar{Q} S_X x_0 \quad (17)$$

$$\Delta U_0 = \Gamma \eta_0 \quad (18)$$

To solve the constrained solution of (15), we can use conventional quadratic programming packages.

4- Stability of the Kautz Method in MPC-LTV

In order to show the stability of the Kautz method in MPC for LTV systems we must show that the solution of (15) equals to the solution of (5) under same conditions. In other words, the Kautz network must be able to capture the optimal solution of (6) precisely. Therefore, we have to study under which conditions the Kautz network is able to capture the required optimal input signals. An infinite series of Kautz functions defines a complete set on the z -domain, and any stable transfer function $G(z)$, can be expressed by the Kautz functions as [18]:

$$G(z) = \sum_{q=1}^{\infty} c_q \Omega_q(z), \quad (19.a)$$

where the c_q 's are called the Kautz coefficients. The corresponding representation of (19.a) in the time domain is

$$g(k) = \sum_{q=1}^{\infty} c_q l_q(k). \quad (19.b)$$

It is shown in [19] that the necessary and sufficient condition for the completeness of the set $\Omega_q(k)$ in (10), (19.a) on $l_2[0, \infty)$ is

$$\sum_{k=0}^{\infty} (1 - |\xi|) = \infty \quad (20)$$

Equation (20) means that a finite-energy signal can be approximated with any degree of accuracy by a linear combination of a finite number of Kautz functions. Worded in another way, Equation (20) means that for linear-stable systems, the coefficients of the high order Kautz networks tend to zero, and the system can then be represented by a finite set of Kautz functions. Therefore, in (14), for each j there exists a finite N_j that synthesises the corresponding control signal precisely.

Orthonormality of the Kautz network in the time domain has been proven in [18], and can be expressed as

$$\sum_{k=0}^{\infty} l_q(k) l_p(k) = \delta(q - p) \quad , \quad q, p = 1, \dots, N \quad (21)$$

Where $\delta(\cdot)$ is the Dirac delta-function. In Kautz-based MPC, a truncated version of (21) is employed by replacing ∞ with the prediction horizon N_p . The necessary prediction horizon to achieve orthonormality of a Kautz network is dependent on its poles, ξ, ξ^* , as well as the network dimension N [5]. As the poles ξ, ξ^* approach the unit circle, a larger prediction horizon N_p is required to maintain the orthonormality. In a similar manner for larger N we need larger N_p to preserve this characteristic [5].

The authors of [12] used the optimal cost function of (5) as a Lyapunov function and terminal equality constraint $x(N_p) = 0$ to provide the general conditions under which stability of model predictive control of a constrained time-varying nonlinear discrete-time system is guaranteed. Under the assumptions of uniform controllability and uniform observability, it is shown there that the value of the optimal cost function of (5) approaches the value of the infinite horizon problem as the prediction horizon N_p approaches infinity. We adopt the same assumptions and follow the results of [5] to establish stability of Kautz parametrization of LTV-MPC. For the sake of simplicity, the stability is established for the case of single-input systems. The extension to multi-input systems is straightforward.

Assumption 1. The final state of the receding horizon optimization problem (15) is constrained to be zero, $x(k + N_p) = 0$, where the terminal state $x(k + N_p)$ is obtained from applying the control sequence $u(k + m) = L(m)^T \eta$, $m = 0, 1, 2, \dots, N_p$, on system (1). We also assume that N_p is large enough so that $u(k + N_p - 1) = 0$.

Assumption 2. For each sampling instant, k , there exists a solution such that the cost function J in (15) is minimized and is equal to the minimized value of J in (6).

Now we are ready to establish the stability of the Kautz parametrisation method for MPC of linear time-varying systems.

Theorem 1. Under Assumptions 1 and 2, the optimization problem of (15) produces an asymptotically stabilising controller for the linear time varying system (1).

Proof. The Lyapunov function for the system (1) is the minimum of the cost function in (15), given as

$$V(x(k),k) = \sum_{m=1}^{N_p} x_0(k+m|k)^T Q x_0(k+m|k) + \sum_{m=1}^{N_p-1} \Delta u_0(k+m|k)^T R \Delta u_0(k+m|k) \quad (22)$$

Where $x_0(k+m|k) = \Phi(k+m, m-1)x_0 + \sum_{\tau=k}^{k+m} \Phi(k+m, \tau+m) BL(m)^T \eta_0$, is the optimizer vector of (15) subject to the constraints. Because $V(x(k),k)$ is a quadratic function and it is positive definite, it follows that it tends to infinity as $x(k)$ tends to infinity. In addition, uniform controllability of (1) ensures us that all the elements of system matrix and input matrix remain bounded over the prediction horizon, so the Lyapunov function (22) is bounded from above over time. The same argument can be made for the Lyapunov function at time $k+1$, that is $V(x(k+1),k+1)$. In order to show the monotonic decreasing of the Lyapunov function we can write

$$V(x(k+1),k+1) - V(x(k),k) = -x_0(k)^T Q x_0(k) - \eta_0^T L(0)^T R L(0) \eta_0. \quad (23)$$

Equation (23) is due to the fact that the principle of optimality (Assumption 2) provides us with the $x_0(k+m|k) = x_0(k+m+1|k+1)$, $u_0(k+m|k) = u_0(k+m+1|k+1)$, so all middle terms of $V(x(k+1),k+1)$ and $V(x(k),k)$ cancel each other. For the final terms of $V(x(k+1),k+1)$, $V(x(k),k)$ we use Assumption 1, hence the asymptotic stability is established.

In the proof of the theorem, we assumed that the Kautz network achieves at each time step the same optimal cost as the conventional MPC method. However, as we will illustrate in the example we are able to achieve stability without achieving optimality. Thus, from a practical point of view, we find that this assumption is usually unnecessary for stability and it could be relaxed. A second line of work is to relax the optimality assumption, and use suboptimal methods to establish stability for the case of LTV systems.

5- Simulation Example

The example presented here illustrate the effectiveness of Kautz networks in model predictive control of linear time-varying systems. The example considers a single input system and compares the solution of Kautz method (Equation (15)) with the solution of traditional MPC, Equation (6).

Consider the linear system with a time-varying unstable pole that is given by

$$x(k+1) = \begin{bmatrix} 1.05 + 0.2 \cos(0.4k) & 2 \\ 0 & 0.9 + 0.3 \sin(0.3k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.08 \end{bmatrix} u(k), \quad (24)$$

The weighting matrices for this system are taken $Q = P = 13I_{2 \times 2}$, $R = 0.1$. The prediction horizon is $N_p = 16$, and the constraint set for the control and states are $|u| \leq 30$, $|\Delta u| \leq 15$, $\|x\|_\infty \leq 8$.

We compare the effect of network dimension and pole value on the behavior of the solution in Figure 3 to Figure 5. In these figures the black-solid line shows the global optimal solution of Equation (6). This solution is achieved with $N = 10$ and an arbitrary value of pole, $|\xi \cdot \xi^*| < 1$, in Equation (15). The near optimum solution is drawn in the figures with the green-dashed line that is achieved with $N = 8$; $\xi = 0.5 + j0.15$. These two lines captured the optimum solution of system (24) perfectly. However, they do not have a satisfactory dimension network as N is too high. The simulation shows that for small values of N the Kautz network is still able to achieve stability. Blue, red and cyan lines are obtained for $N = 2$ that is the smallest possible value (the dimension of the network in Kautz parametrization must be even). The difference between these lines is due to the different selection of pole value. The blue and red lines converge to the origin, though in a different manner compared to the optimal solution. However, the cyan-dotted line is trapped in an oscillation which shows instability. These results illustrate that the effect of network dimension is similar to the effect of control horizon, a larger N tends to a closer behavior to the optimum solution. Additionally, the value of ξ is not very influential for large N (as we see for $N = 10$). The difference between red, blue and cyan lines indicates that as N decreases, the value of ξ becomes important in the terms of stability and optimality.

The figures also demonstrate that the optimality of the solution is not necessary for achieving stability. This result allows us to use small values on N and reduce computation times. However, during the stability proof (Theorem 1) we used the optimality of the Kautz network. The findings here suggest us to relax the assumptions of the theorem to allow suboptimal solutions as well as the optimal one. This direction of study is the subject of our ongoing research.

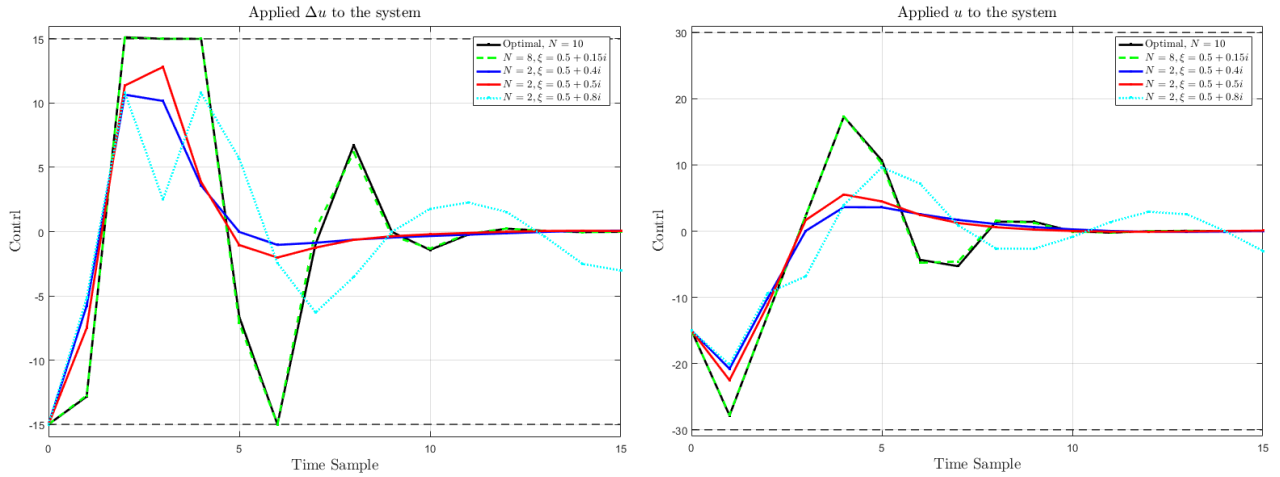


Figure 3. Applied input for different Kautz Characteristics

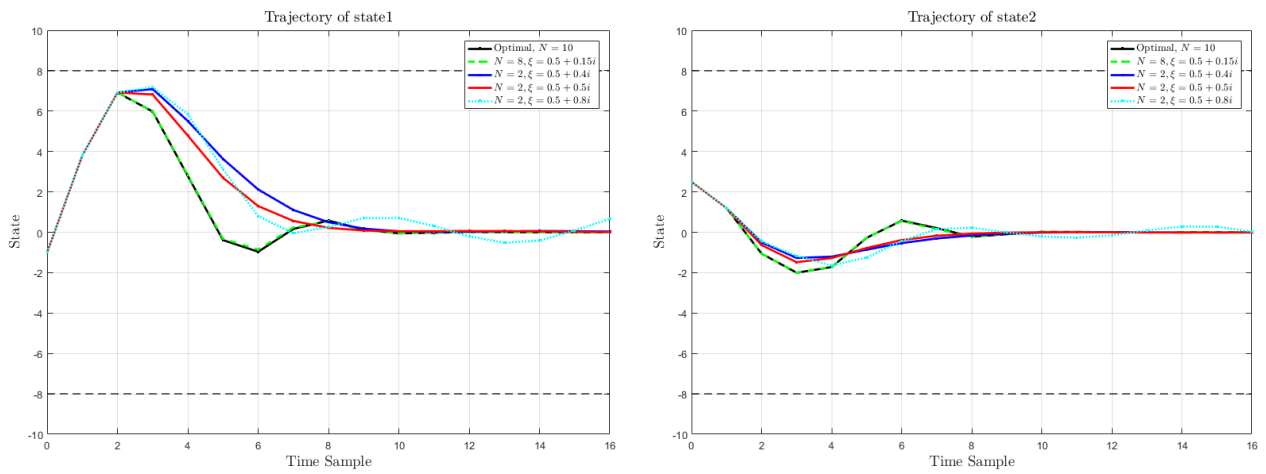


Figure 4. State evolution for different Kautz Characteristics

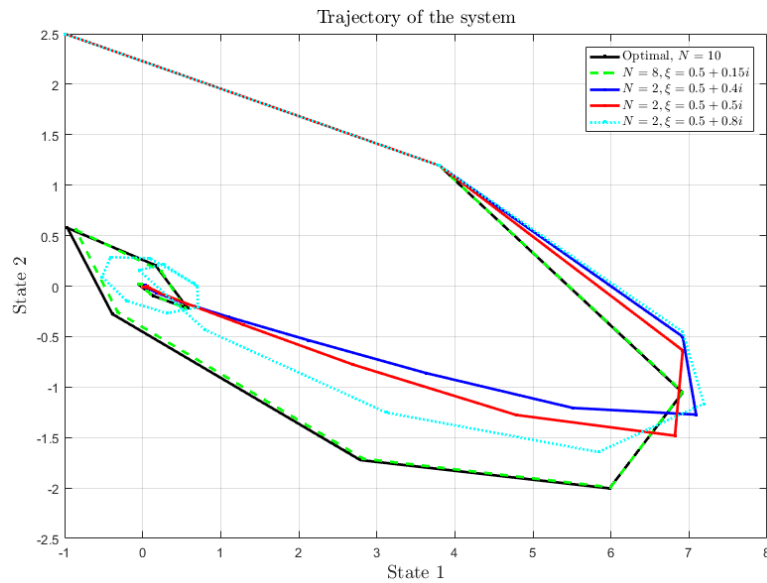


Figure 5. Phase plane trajectory for different Kautz Characteristics

6- Conclusion

The problem of applying Kautz parametrization to the model predictive control of linear time varying systems has

been addressed in this paper. It was demonstrated that by means of a Kautz parametrization we can express the future control signals of a time-varying system by a smaller number of decision variables compared to the traditional method. Consequently, the required computation times for solving the optimization problem are decreased as the total number of decision variables decreases, and this constitutes the main advantage of the Kautz parametrization. The stability of Kautz parametrization for MPC of linear time-varying systems was established by means of optimality arguments. In the simulation example, it was verified that even with very small values of N , the Kautz parametrization method preserves stability, which will prompt us to explore the use of suboptimal methods to establish the stability of the scheme. Also, the effects of active constraints, pole location and network degree on the performance of the Kautz-based MPC method were studied through the simulations.

7- References

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