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Effective Forecasting of Insurer Capital Requirements: ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH Approaches

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Abstract

This research paper presents a comprehensive analysis of three prominent volatility and dependence models for financial time series: ARMA-GARCH, GARCH-EVT, and DCC-GARCH. These models are employed to assess and forecast capital requirements for life and non-life insurer investments. This study evaluates the models' performance in forecasting Value-at-Risk, using daily data on key Thai financial indicators (representing permissible insurer investment assets) from March 2009 to March 2024. Specifically, 1-day and 10-day VaR forecasts are generated using the ARMA-GARCH and DCC-GARCH models, while the ARMA-GARCH-EVT model is employed for 1-day VaR forecasting. Our findings indicate that the ARMA-GARCH model effectively captures time-varying volatility, while the GARCH-EVT approach enhances tail risk estimation, particularly relevant for stress testing. Additionally, the DCC-GARCH model allows for the examination of dynamic conditional correlations between assets, providing insights into portfolio diversification benefits. Rigorous backtesting procedures, employing Kupiec and Christoffersen tests with a rolling window of 1,000 out-of-sample observations, confirm that the majority of models accurately forecast VaR at their respective horizons, with only a very small subset of 10-day VaR models exhibiting limitations. These results highlight that ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH models offer insurers robust tools for estimating minimum capital requirements, forecasting investment risk, and guiding strategic asset allocation decisions. This research underscores the effectiveness of these models for practical application in the insurance industry while also emphasizing the importance of continued model validation, particularly for extended forecasting horizons.

Keywords:

Volatility Forecasting;

GARCH Models;

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Insurance Risk Management;

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1- Introduction

Insurance companies are crucial to the financial sector's stability, enabling them to adeptly manage risks and uphold their commitments to policyholders. Solvency, underpinned by a stringent regulatory framework that stipulates minimum capital requirements, is vital for their continuous operation and the health of the industry, as affirmed by scholars like [1-3]. The International Association of Insurance Supervisors (IAIS) promotes these principles globally through the Insurance Core Principles (ICPs), which establish standards for capital adequacy and guide national regulators, as noted by Gaganis et al. (2016) [4].

In Thailand, the Office of Insurance Commission (OIC) diligently enforces capital requirements in line with the IAIS' ICPs, thereby bolstering the financial system's stability and curtailing insolvency risks. The OIC's policies, detailed in official notifications, emphasize the significance of investment diversification for insurers. Firms are encouraged to

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spread their capital across various asset classes such as equities, commodities, real estate, fixed-income securities, and foreign currencies in order to diffuse risk concentration. Capital charges set by the OIC are calibrated based on the risk profile of each investment asset class, fortifying insurers' solvency in both regular and stressed market conditions. These regulatory precautions are crafted to shield against potential investment shortfalls, aiding insurers in handling market fluctuations, safeguarding customer interests, and maintaining substantial capital reserves.

In the realm of financial risk management, the complexity of market dynamics necessitates a shift from conventional analytics to advanced econometric methodologies. Traditional models, such as simple moving averages or historical simulation, often relying on an oversimplified assumption of constant variability, fail to adequately capture the complex stochastic behavior of financial market returns, particularly during unforeseen market events [5-7]. This inadequacy is particularly evident in the presence of heteroskedasticity, where volatility clusters and exhibits variability across different time periods, highlighting the need for more sophisticated risk assessment tools [8]. Zhou et al. (2021) [5], Huang et al. (2024) [6], and Petkov et al. (2021) [5-7] further emphasized how traditional approaches struggle to accurately model tail risks, leaving institutions vulnerable to extreme market movements. While the potential benefits of both Autoregressive Moving Average and Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) models existed before the 2008 financial crisis, they were not fully realized, especially when used in combination. Had they been applied more rigorously, these models could have provided earlier warnings of escalating risks, potentially enabling more effective interventions such as forecasting more appropriate capital requirements for insurers.

The 2008 crisis underscored the need for models that could better account for volatility clustering, leading to increased interest in GARCH-type approaches. Regulatory bodies and insurance entities have since begun to adopt more robust econometric designs, such as ARMA-GARCH models, within their risk management strategies. Numerous studies have demonstrated the superior performance of GARCH models in forecasting volatility across various asset classes, including equities, bonds, and commodities [9-15]. For example, research on the Indian stock market highlights the ability of GARCH models to identify temporal patterns in market trends and forecast volatility with greater accuracy. These findings suggest a predictable behavior of market turbulences over intervals, indicating that GARCH-based approaches can effectively model these patterns.

While GARCH models have proven effective in capturing volatility clustering across various financial markets, their application to forecasting short- and long-term Value-at-Risk (VaR) at both individual asset and portfolio levels requires further exploration. This study addresses this gap by first employing an ARMA-GARCH framework to model volatility dynamics and forecast one-day and ten-day VaR for several key asset classes: equities, commodities, real estate, fixed-income securities, and foreign currencies. As a widely used risk metric, VaR quantifies potential losses over specific time horizons and confidence levels, proving crucial for informed risk management [16]. For each asset class, this study utilizes an ARMA model to capture the conditional mean dynamics and a GARCH model to estimate the time-varying volatility, allowing for a comprehensive assessment of individual asset risk exposures. Our approach employs a rolling window on out-of-sample data to enhance the robustness and timeliness of these single-asset VaR estimates. The one-day and ten-day VaR forecasts offer insights into short-term trading and strategic asset allocation, respectively, with the latter derived from Bollerslev's ARCH paradigm [17]. The preference for the GARCH model stems from its effectiveness in capturing volatility clustering in financial time series [6, 18-27]. However, to further enhance risk assessment, particularly in the tails of the distribution, this study also incorporates GARCH-EVT, which leverages extreme value theory. This study further employs a portfolio-level analysis using Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) to capture asset class co-movements and provide a comprehensive portfolio VaR analysis. By leveraging EVT, this advanced approach aligns with the risk management protocols of the Basel Accords, providing sophisticated methods to calculate the necessary capital for market risk. Supported by theoretical and empirical underpinnings [28-30], this study contributes significantly to risk management practices by offering refined methods to accurately estimate minimum capital requirements for both one-day and ten-day periods.

This study elevates the modeling of financial market volatility through the integration of ARMA-GARCH with Extreme Value Theory (EVT), acknowledging the importance of capturing tail risks. The predictive power of EVT is well-established, allowing for the extrapolation of risks from events that are not bounded by historical precedent [31-34]. The ARMA-GARCH-EVT model effectively estimates the probability and impact of rare market events, enhancing traditional volatility analysis and strengthening stress testing capabilities [35-37]. The integrated model takes advantage of GARCH residuals being independently and identically distributed, providing a robust framework for risk evaluation [38-41]. This sophisticated approach to risk measurement bolsters stress testing capabilities, mitigating the risk of capital shortfalls and reputational risks due to underestimating market risk [42]. Our application of a rolling window analysis furthers the precision of one-day VaR calculations, a critical factor in establishing insurer capital reserves against significant market downturns, an aspect vital as identified by Brooks & Persand (2003) [43]. This methodology not only meets but anticipates regulatory standards, ensuring capital adequacy and financial resilience in the face of potential investment losses.

During periods of market volatility, understanding variable asset correlations is pivotal, as corroborated by a breadth of research [44-48]. Building on the seminal works of Engle & Sheppard (2001) [49] and Engle (2002) [50], these studies utilized DCC-GARCH model to adeptly trace evolving risk profiles through the dynamics of co-volatility. This insight is critical for comprehensive risk exposure management and maintaining diversified portfolios. The robustness of the DCC-GARCH model in volatile markets is well-supported by research studies [51-59]. Employing a rolling-window approach, our analysis forecasts out-of-sample portfolio performance, updating VaR estimates to mirror current market trends. For life and non-life insurers, this methodology proves invaluable, enabling them to synchronize asset portfolios with liability timelines and enhance their risk management practices. Moreover, such strategies allow for the recalibration of investment positions to maintain adequate capital buffers, adhering to the regulatory standards set by the OIC. Our research equips insurance firms with an advanced risk assessment tool that embraces the intricacies of asset correlation over time, which is especially useful in the event of financial disruptions akin to the 2008 crisis. By foreseeing risk correlations' upsurge, the DCC-GARCH model empowers insurers to proactively adjust their strategies, positioning them to better withstand future market stresses.

To effectively navigate this paper, it is important to articulate the specific objectives that guided our analysis. The primary aims of this research are threefold: first, to apply the ARMA-GARCH and ARMA-GARCH-EVT models to improve the accuracy of VaR calculations integral to insurance capital reserve legislation; second, to leverage the DCC-GARCH model to examine the interplay between asset correlations and market volatility, with a focus on portfolio diversification strategies; and third, to evaluate the efficacy of these models through rigorous backtesting procedures. Of particular novelty in our approach is the integration of EVT with the ARMA-GARCH model to enhance stress testing in volatile markets. Additionally, our study pioneers the application of these advanced econometric models within the regulatory framework specific to Thailand's Office of Insurance Commission, thus providing a valuable template for risk assessment in emerging markets. By setting clear objectives and highlighting these novel contributions, this paper aims to advance the field of financial risk management by offering implementable strategies for insurance firms to strengthen financial solvency and resilience.

The paper is organized as follows: Section 2 presents the dataset, describing its sources and characteristics, and the methodology, including the ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH modeling techniques utilized to estimate asset volatility. Additionally, the backtesting approach is presented here. Section 3 presents the main empirical analysis, which applies the models to the data and discusses the implications of our findings, examining the models' performance and potential impact within the field. Finally, Section 4 concludes with a summary of the findings, outlining the contributions of this study to financial risk assessment and suggesting avenues for future research.

2- Domain of Experiment and Methodology

Daily returns, calculated using the logarithmic return formula ($r_t = \ln(P_t/P_{t-1})$), were analyzed for a dataset comprising 3,853 observations from March 1, 2009, to March 29, 2024. The dataset encompasses key Thai financial indicators: the SET index, Brent crude oil prices, government bond prices (3-7 and 7-10 year maturities), the JPY/THB exchange rate, and the property development sector index. Data were obtained from Datastream International and Bloomberg. Descriptive statistics, presented in Table 1, reveal characteristic features of financial time series. For instance, with the exception of the JPY/THB exchange rate, all series exhibit negative skewness, indicating a higher probability of large negative returns compared to a normal distribution. Additionally, all series display leptokurtosis, evidenced by high kurtosis values, implying a greater concentration of observations around the mean and fatter tails than a normal distribution. This leptokurtic behavior suggests an increased likelihood of extreme return movements. Volatility clustering is also evident, as confirmed by the Jarque-Bera (JB) test. The Augmented Dickey-Fuller (ADF) test confirms that all series are stationary, justifying the use of time-series models. Notably, heightened volatility during 2020 underscores the importance of incorporating dynamic volatility patterns into risk management, especially during periods of market disruption.

Table 1. Summary statistics for six assets

	SET Index	Brent Crude Oil	GOV2 3-7 TTM	GOV3 7-10 TTM	JPY/THB	Property Index
Mean	0.00043	0.00015	0.00011	0.00014	-0.00010	0.00050
Median	0.00027	0.00072	0.00013	0.00015	-0.00012	0.00014
Maximum	0.07656	0.20340	0.00859	0.01651	0.04207	0.08324
Minimum	-0.11384	-0.27976	-0.00947	-0.01768	-0.03602	-0.14303
SD	0.01012	0.02291	0.00121	0.00242	0.00594	0.01308
Skewness	-0.87236	-0.71476	-0.60515	-0.37548	0.11484	-0.78790
Kurtosis	12.51147	17.13661	9.19053	6.083446	3.877468	10.40968
JB	25,652	47,531	13,814	6,041	2,427	17,819
JB (Probability)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ADF	-12.267	-12.521	-11.894	-12.287	-13.756	-11.144
ADF (Probability)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Note: This table presents summary statistics for the daily return series of six assets. There are a total of 3,853 returns for each asset.
The Jarque-Bera (JB) and Augmented Dickey-Fuller (ADF) test results are statistically significant at 0.01.

The analysis in this paper employs three complementary approaches to model asset volatility and risk which are as follows:

2-1- Modeling Dynamic Volatility using Autoregressive Moving Average and Generalized Autoregressive Conditional Heteroskedastic

Financial time series data often exhibit complex patterns, including volatility clustering and autocorrelation. To capture these complexities, this study utilizes Autoregressive Moving Average and Generalized Autoregressive Conditional Heteroskedastic (ARMA-GARCH) models. GARCH models are particularly well-suited for analyzing and forecasting volatility, which is a crucial component of financial risk management. Incorporating an ARMA model further enhances this analysis by explicitly modeling the conditional mean of the return series. The following equations describe the ARMA(*m*, *n*)-GARCH(*p*, *q*) model used in this study:

$$\begin{cases} r_t = \mu + \sum_{i=1}^{m} \emptyset_i \, r_{t-i} + \sum_{j=1}^{n} \theta_j \, \varepsilon_{t-j} + \varepsilon_t \\ \varepsilon_t = \sigma_t z_t \\ \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2 \end{cases} \tag{1}$$

where r_t represents the return at time t, μ is the constant term in the mean equation, $\emptyset_i$ represents the parameters of the AR(m) model and θ_j represents the parameters of the MA(n) model, ε_t is the residual of the mean equation at time t, σ_t^2 is the conditional variance at time t, z_t represents the standardized innovations (or shocks) to the volatility process and is a sequence of independently and identically distributed (i.i.d.) random variables with a standard normal distribution with zero mean and unit variance, and ω is the constant term in the variance equation. The α_i terms capture how sensitive current volatility is to recent shocks, while the β_j terms measure the persistence of volatility from past periods. A larger α_i implies that recent shocks have a stronger effect on today's volatility, indicating a market that is more sensitive to new information. Similarly, a larger β_j suggests that volatility shocks tend to linger, implying a market where volatility clusters and periods of high/low volatility tend to persist. The sum ($\alpha + \beta$) is often used as a measure of overall volatility persistence, with values close to 1 indicating long-lasting volatility shocks. To ensure the stationarity of the variance process, the parameters must satisfy the conditions: $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

To identify the most appropriate ARMA-GARCH specification, this study considered a range of values for *m*, *n*, *p*, and *q* during the model selection process. Model validity is assessed by examining the residual series for constant mean and variance, as well as the absence of autocorrelation. Given the potential non-normality in the residuals, this GARCH model, with normal innovations, is fitted using the pseudo-maximum likelihood estimation procedure. The Akaike Information Criterion (AIC) is employed to determine the most appropriate ARMA-GARCH specification. As highlighted by Ardia (2008) [60] and Makridakis et al. (2008) [61], the model with the lowest AIC value is considered relatively optimal. Therefore, this paper will select the model exhibiting the lowest AIC among the candidate models. Equation, as defined by Akaike (1981) [62], presents the AIC calculation:

$$\text{AIC} = -2lnL + 2K \tag{2}$$

where *L* represents the likelihood function of the model and *K* denotes the number of estimated parameters in the ARMA-GARCH model.

2-2- Modeling Tails using Extreme Value Theory

Modeling the extreme movements in the tails of the asset return distribution is crucial for accurately assessing financial risk. The Extreme Value Theory (EVT) provides a robust framework for capturing these tail behaviors, which are essential for tasks such as estimating tail risk and setting appropriate capital requirements. Studies by researchers like [16, 41, 63] explored the application of EVT in finance. Building upon the ARMA(*m*, *n*)-GARCH(*p*, *q*) model established earlier, this study leverages the peak over threshold (POT) method to model the tails of the standardized residuals using a Generalized Pareto Distribution (GPD). This approach allows for a more nuanced understanding of extreme events within the context of the identified volatility dynamics.

2-2-1- Peak Over Threshold Model

The Peak Over Threshold (POT) method focuses on the distribution of exceedances over a predetermined threshold u. Let X denote a series of i.i.d. losses with cumulative distribution function $F(x)$. The conditional cumulative distribution function of the excess value $y = X - u$, given that $X > u$, is defined as:

$$F_u(y) = \Pr((X-u) \leq y | X > u) = \frac{F(u+y) - F(u)}{1 - F(u)}, y \geq 0 \tag{3}$$

The goal is to find a suitable parametric distribution to model $F_u(y)$. Balkema & de Haan (1974) [64] and Pickands (1975) [65] showed that for a sufficiently high threshold u, the limiting distribution of these excesses can be well-approximated by the Generalized Pareto Distribution (GPD):

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\beta}\right)^{\frac{-1}{\xi}} & if \quad \xi \neq 0 \\ 1 - e^{\frac{-y}{\beta}} & if \quad \xi = 0 \end{cases} \tag{4}$$

where, ξ and β are the shape and scale parameters of the GPD, respectively. This function embodies three types of distributions. If $\xi > 0$, it corresponds to a heavy-tailed distribution. When $\xi = 0$, it represents an exponential distribution. If $\xi < 0$, it indicates a bounded distribution (sometimes called Pareto type II).

2-2-2- Extreme Value Theory and Estimation of Value-at-Risk

Given that $F_u(y)$ converges to the GPD for sufficiently large u and $X = y + u$ for $X > u$, this study has the following representation:

$$F(x) = [1 - F(u)]F_u(y) + F(u), X > u \tag{5}$$

The tail of the underlying distribution $F(x)$ can be expressed as:

$$F(x) = [1 - F(u)]G_{\xi,\beta}(x - u) + F(u), X > u \tag{6}$$

Therefore, the tail estimator of $F(x)$ is given by:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \hat{\xi}\frac{x-u}{\hat{\beta}}\right)^{-\frac{1}{\hat{\xi}}}, X > u \tag{7}$$

where n is the total number of observations, N_u is the number of observations exceeding the threshold u, and $\hat{\xi}$ and $\hat{\beta}$ are the maximum likelihood estimates of the shape and scale parameters obtained from the excess data, respectively.

For a given probability, $q > F(u)$, the unconditional Value-at-Risk (VaR) quantile is obtained by inverting Equation 7 to get:

$$\text{VaR}_q = u + \frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-q)\right)^{-\hat{\xi}} - 1\right] \tag{8}$$

2-3- Dynamic Conditional Correlation–Generalized Autoregressive Conditional Heteroskedasticity Models

While effective for individual assets, univariate GARCH models are limited in their ability to capture the dynamic relationships between assets. To address this limitation, the Dynamic Conditional Correlation (DCC) model is employed, a multivariate GARCH model that captures both volatilities and time-varying correlations. Unlike the Constant Conditional Correlation (CCC) model, which assumes static correlations, the DCC model allows correlations to fluctuate over time, providing a more realistic representation of market dynamics. This analysis investigates the empirical applicability of the Dynamic Conditional Correlation–Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model in estimating large conditional covariance matrices, allowing for time-varying conditional correlations as proposed by Engle (2002) [50]. The estimation of Engle's DCC-GARCH comprises two steps.

In the first step, a univariate GARCH model is estimated for each asset as follows:

$$\begin{cases} r_t = \mu_t + \varepsilon_t, \ \varepsilon_t | I_{t-1} \sim N(0, H_t) \\ \varepsilon_t = \sqrt{H_t} z_t \end{cases} \tag{9}$$

where r_t represents the $(M \times 1)$ vector of log returns of M assets at time t; μ_t is the $(M \times 1)$ vector of the conditional means of M assets at time t; ε_t is an $(M \times 1)$ vector of the residuals process; I_{t-1} represents the information set at time $t-1$. H_t is the multivariate conditional variance-covariance $(M \times M)$ matrix of ε_t. $\sqrt{H_t}$ is obtained using Cholesky decomposition, and z_t is the $(M \times 1)$ vector of independent and identically distributed random errors such that $E[z_t] = 0$ and $E[z_t z_t'] = I_T$, where I_T denotes the identity matrix of order T.

The second step focuses on specifying a time-varying multivariate conditional variance. The multivariate DCC-GARCH model is then defined as follows:

$$\begin{cases} H_t = D_t\, R_t\, D_t \\ R_t = \left(diag(Q_t)\right)^{-1/2} Q_t \left(diag(Q_t)\right)^{-1/2} \\ D_t = diag\left(\sqrt{h_{11,t}}, \sqrt{h_{22,t}}, \dots, \sqrt{h_{KK,t}}\right) \end{cases} \tag{10}$$

where D_t is a diagonal $(M \times M)$ matrix of conditional standard deviations for return series at time t, obtained from estimating a univariate GARCH model with $\sqrt{h_{ii,t}}$ on the i^{th} diagonal, $i = 1, 2, \dots, M$.

The DCC specification is defined as follows:

$$\begin{cases} Q_t = (1 - \alpha - \beta)S + \alpha z_{t-1}\, z'_{t-1} + \beta Q_{t-1} \\ R_t = Q_t^{*\,-1} Q_t\, Q_t^{*\,-1} \end{cases} \tag{11}$$

where $Q_t = \left[q_{ij,t}\right]$ is $(M \times M)$ time-varying covariance matrix of the standardized residuals, z_t. S is the $(M \times M)$ unconditional covariance matrix of the standardized residuals, z_t, and α and β are non-negative scalar parameters that satisfy $\alpha + \beta < 1$. $Q_t^* = diag(\sqrt{q_{ii,t}})$ is a diagonal matrix with the square root of the element of Q_t on its ith diagonal position. R_t is the symmetric dynamic conditional correlation $(M \times M)$ matrix with $\rho_{ij,t} \leq 1$ and $\rho_{ii,t} = 1$. The conditional correlation $\rho_{ij,t} = q_{ij,t}/\sqrt{q_{ii,t} q_{jj,t}}$ are the elements in the matrix R_t that is positive definite.

As noted by Engle (2002) [50], the DCC model could be estimated by using a two-step approach to maximize the log-likelihood function. Let θ denote the parameters in D_t and φ the parameters in R_t. Under the Gaussian assumption, the log-likelihood can be decomposed as follows:

$$L_t(\theta, \varphi) = \left[-\frac{1}{2} \sum_{t=1}^{T} n log(2\pi) + log|D_t|^2 + \varepsilon'_t D_t^{-2} \varepsilon_t\right] + \left[-\frac{1}{2} \sum_{t=1}^{T} log|R_t| + z'_t R_t^{-1} z_t - z'_t z_t\right] \tag{12}$$

2-4- Forecasting Method

A rolling window forecasting method generated out-of-sample predictions. Initially, a window comprising the first 2,853 observations (approximately 11 years of daily trading data) was used to estimate the parameters of the ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH models. To generate forecasts, the estimation window was then rolled forward by one observation at a time. This process involved removing the earliest observation and adding the next one from the dataset, maintaining a constant window size of 2,853. Re-estimation and forecasting were repeated sequentially, with each iteration producing a one-step-ahead forecast. For the ARMA-GARCH and ARMA-GARCH-EVT models, this yielded a forecast of the conditional mean, μ_{t+1}, and conditional variance, σ^2_{t+1}. The DCC-GARCH model produced forecasts of the conditional mean vector, r_{t+1}, and the conditional covariance matrix, H_{t+1}, capturing the dynamic correlations among the assets in the portfolio. Ultimately, this procedure yielded a total of 1,000 out-of-sample forecasts for each model, covering the period from observation 2,854 to 3,853.

2-5- Value-at-Risk Measures

Building upon the ARMA-GARCH and ARMA-GARCH-EVT models for volatility forecasting, this section details the calculation of Value-at-Risk (VaR) measures. These measures provide a forward-looking assessment of potential portfolio losses and inform capital adequacy decisions.

2-5-1- ARMA-GARCH VaR

Capital requirements for market risk are determined using the VaR measure. Given the one-step-ahead forecasts of the conditional mean, μ_{t+1}, and conditional variance, σ^2_{t+1}, from the ARMA-GARCH model, the 1-day VaR at time $t + 1$ with a confidence level of $(1 - \alpha)$ can be calculated as:

$$\mathrm{VaR}^{\alpha}_{t+1} = \mu_{t+1} + Z_\alpha \sigma_{t+1} \tag{13}$$

where Z_α is the α quantile of the standard normal distribution. A 95% confidence level aligns with Thai regulations for calculating the 1-day VaR. For the 10-day VaR, which follows the Basel requirements at a 99% confidence level, a rolling window approach generates a sequence of 10 one-step-ahead forecasts from the ARMA-GARCH model. The 10-day VaR is then calculated as the 1st percentile of the simulated 10-day portfolio loss distribution.

2-5-2- ARMA-GARCH-EVT VaR

To assess the capital requirement under a stress scenario, the ARMA-GARCH-EVT model estimates the 1-day VaR at time $t + 1$ with a 97.5% confidence level. This approach leverages the EVT framework to better capture the tail risk of asset returns, which is particularly relevant during periods of market stress. The 1-day VaR of ARMA-GARCH-EVT at time $t + 1$, derived from the conditional EVT, can be expressed as:

$$\mathrm{VaR}_{t+1}^{\alpha} = \mu_{t+1} + \sigma_{t+1}\left[u\frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-q)\right)^{-\hat{\xi}} - 1\right]\right] \tag{14}$$

where the one-step-ahead forecasts of the conditional mean, μ_{t+1}, and conditional variance, σ_{t+1}^2, from the ARMA-GARCH model, n is the total number of observations, N_u is the number of observations exceeding the threshold u, and $\hat{\xi}$ and $\hat{\beta}$ are the maximum likelihood estimates of the shape and scale parameters of GPD, respectively.

2-5-3- DCC-GARCH VaR

Accurate estimation of portfolio risk is crucial for determining adequate capital reserves. The DCC-GARCH model is leveraged to forecast the 1-day VaR at time $t+1$ for a portfolio of assets. The DCC-GARCH model captures dynamic correlations among assets by forecasting both the conditional mean vector, μ_{t+1}, and the conditional covariance matrix, H_{t+1}. Under the assumption of normality and a confidence level of $(1-\alpha)$, the 1-day VaR can be expressed as:

$$\mathrm{VaR}_{t+1}^{\alpha} = w'\mu_{t+1} + Z_{\alpha}\sqrt{w'H_{t+1}w} \tag{15}$$

where w is the weight vector of the assets in the portfolio and Z_{α} is the α quantile of the standard normal distribution. This study specifically employs the DCC-GARCH model to estimate the 1-day VaR at a 95% confidence level, aligning with Thai regulations. For a 10-day VaR, adhering to Basel requirements at a 99% confidence level, a rolling window approach generates a sequence of 10 one-step-ahead forecasts from the DCC-GARCH model.

Two investment portfolios are constructed--one tailored for life insurers and one for non-life insurers--guided by two key objectives: duration matching, aligning the portfolio duration with the specific liability structures of each insurer type, and VaR minimization, employing the DCC-GARCH model to determine portfolio allocations that minimize the estimated VaR, thereby reducing the capital required to cover potential losses. This focus on minimizing risk, while incorporating duration matching, aligns with the core principles of Markowitz portfolio theory [66], which emphasizes the trade-off between risk and return in portfolio allocation. In determining the optimal asset allocation proportions, investment weight limits stipulated by the Thai OIC are adhered to: 1) stocks, exchange rates, and property each have a maximum allocation of 30%, 2) crude oil, representing the broader commodity asset class, has a maximum allocation of 5%, and 3) bonds have no regulatory limit on allocation. These constraints, along with the conditions that $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$, where w_i is the weight of the i^{th} asset, are incorporated into the optimization process. This incorporation ensures that the resulting portfolios comply with regulatory requirements while striving to achieve the dual objectives of duration matching and VaR minimization.

2-6- Backtesting

Backtesting is essential for evaluating the accuracy of VaR models by comparing predicted VaR with actual losses. Two widely recognized backtesting methods are employed: Kupiec's unconditional coverage test [67] and Christoffersen's conditional coverage test [68].

2-6-1- Kupiec's Unconditional Coverage Test

Kupiec's unconditional coverage test focuses on the frequency of exceedances, instances where actual losses exceed the predicted VaR. The null hypothesis (H_0) states that the observed frequency of exceedances is statistically consistent with the expected frequency based on the model's chosen confidence level. Let N be the number of exceedances over T trading days, and α be the probability of an exceedance. Under H_0, N follows the binomial distribution with parameters (T, α). The likelihood ratio test statistic is:

$$LR_{UC} = -2\ln\left[\frac{(1-\alpha)^{T-N}(\alpha)^{N}}{\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}}\right] \sim \chi_1^2 \tag{16}$$

If the calculated LR_{UC} exceeds the critical value from the chi-squared distribution with 1 degree of freedom at a chosen significance level (e.g., 5%), this study rejects H_0, indicating the model's predicted exceedance frequency is inaccurate.

2-6-2- Christoffersen's Conditional Coverage Test

While Kupiec's test assesses the overall frequency of exceedances, Christoffersen's test goes a step further by examining whether these exceedances are independent over time. The null hypothesis (H_0) for Christoffersen's test is that the exceedances are independent, meaning an exceedance on one day does not affect the probability of an exceedance on the following day. Let n_{ij} be the number of observations where transition i, j occurs, representing the four possible combinations of exceedances (denoted as 1) or no exceedances (denoted as 0) on consecutive days. The following conditional probabilities can then be defined:

$\pi_{01} = P(I_{t+1} = 1 | I_t = 0) = P(Exceedance\ tomorrow\ |\ No\ exceedance\ today)$
$\pi_{11} = P(I_{t+1} = 1 | I_t = 1) = P(Exceedance\ tomorrow\ |\ Exceedance\ today)$

under the assumption of independence, π_{01} should equal π_{11}, and both should equal to the model's specified probability of an exceedance, α. Christoffersen's test uses a likelihood ratio statistic to compare the likelihood of the observed data under independence versus without this restriction:

$$LR_{CC} = 2ln\left[(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\right] - 2ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] \sim \chi_2^2 \quad (17)$$

where N represents the total number of observed exceedances over T trading days. Similar to Kupiec's test, if the calculated LR_{CC} statistic exceeds the critical value from the chi-squared distribution with 2 degrees of freedom at a chosen significance level, the null hypothesis of independence is rejected. This suggests that the model might not adequately capture the time-varying nature of risk.

The research methodology procedures are visually represented in Figure 1.

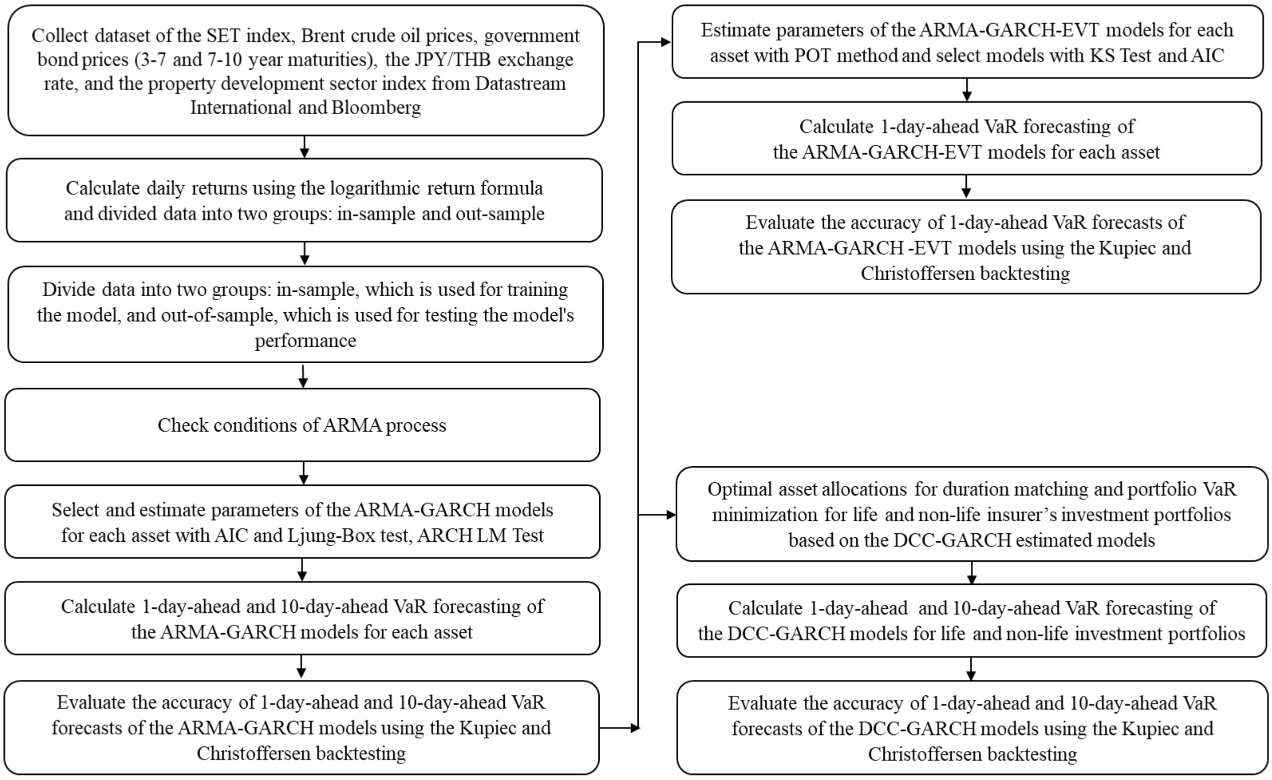


Figure 1. Procedures for the research methodology

3- Empirical Results

This section presents the empirical findings of the study, beginning with the estimation results of the ARMA-GARCH models for each asset. Subsequently, the analysis extends to the ARMA-GARCH-EVT model, which incorporates extreme value theory to capture tail risk. Finally, the DCC-GARCH model is employed to assess the dynamic correlations among assets and their impact on portfolio VaR. The results of each model are discussed in detail, highlighting key findings and their implications for risk management within the Thai insurance industry.

3-1- ARMA-GARCH Estimation

Table 2 presents the in-sample parameter estimates of the selected ARMA-GARCH model specifications for each asset based on the AIC. The selected models employed distinct orders for different assets: for example, ARMA(3,3)-GARCH(1,1) for the SET index, ARMA(3,2)-GARCH(1,1) for Brent crude oil, ARMA(1,1)-GARCH(1,1) for government bond prices (3-7 year maturities), ARMA(2,1)-GARCH(1,1) for government bond prices (7-10 year maturities), ARMA(2,0)-GARCH(1,1) for the JPY/THB exchange rate, and ARMA(1,1)-GARCH(1,1) for the property development sector index. Notably, all assets exhibit persistent volatility clustering, as indicated by the near-unity sum of the ARCH and GARCH coefficients across the selected models. This finding underscores the importance of incorporating time-varying volatility into risk management models for all assets included in this analysis. The significant and positive coefficients of lagged squared returns further emphasize the presence of strong GARCH effects, suggesting that historical volatility information is crucial for predicting future volatility. These results are consistent with findings by Floros (2007) [69], who observed similar evidence of persistent volatility in mature and emerging markets. Our study extends these findings by demonstrating the relevance of GARCH models for capturing volatility dynamics specifically within the context of the investment risk of the Thai insurance industry. Diagnostic tests, including the Ljung-Box test on standardized squared residuals and the ARCH-LM test, confirm the absence of autocorrelation and autoregressive conditional heteroskedasticity (p-values > 0.05 for most assets, except for Brent crude oil, where the p-value is slightly above 0.01). However, these results still generally support the adequacy of the selected GARCH models in capturing volatility dynamics. The robust performance of the selected GARCH models highlights their value as a tool for risk management and capital adequacy assessment within the Thai insurance sector.

Table 2. Parameter estimation results of the ARMA-GARCH model

Model	SET Index ARMA(3,3)-GARCH(1,1)	Brent Crude Oil ARMA(3,2)-GARCH(1,1)	GOV2 3-7 TTM ARMA(1,1)-GARCH(1,1)	GOV3 7-10 TM ARMA(2,1)-GARCH(1,1)	JPY/THB ARMA(2,0)-GARCH(1,1)	Property Index ARMA(1,1)-GARCH(1,1)
Mu (p-value)	0.0006 (0.0001***)	0.0002 (0.3601)	0.0001 (0.0000***)	0.0001 (0.0000***)	-0.0001 (0.1885)	0.0006 (0.0102**)
ar(1) (p-value)	0.4916 (0.0000***)	0.2065 (0.0000***)	0.2911 (0.0000***)	-0.480754 (0.0049***)	-0.0272 (0.1737)	0.9373 (0.0000***)
ar(2) (p-value)	0.7415 (0.0000***)	-0.9890 (0.0000***)	-	0.2549 (0.0000***)	0.0055 (0.7796)	-
ar(3) (p-value)	-0.7430 (0.0000***)	-0.0368 (0.0000***)	-	-	-	-
ma(1) (p-value)	-0.4515 (0.0000***)	-0.2426 (0.0000***)	0.0573 (0.2992)	0.7812 (0.0000***)	-	-0.9103 (0.0000***)
ma(2) (p-value)	-0.7405 (0.0000***)	1.0008 (0.0000***)	-	-	-	-
ma(3) (p-value)	0.7181 (0.0000***)	-	-	-	-	-
omega (p-value)	7.92E-07 (0.4494)	3.15E-06 (0.5498)	1.69E-08 (0.9307)	1.01E-07 (0.7493)	1.00E-06 (0.2220)	2.00E-06 (0.1535)
alpha1 (p-value)	0.1021 (0.0000***)	0.0791 (0.0129**)	0.0807 (0.0000***)	0.1137 (0.0000***)	0.0739 (0.0000***)	0.0913 (0.0000***)
beta1 (p-value)	0.8968 (0.0000***)	0.9191 (0.0000***)	0.9064 (0.0000***)	0.8787 (0.0000***)	0.9132 (0.0000***)	0.0930 (0.0000***)
AIC	-6.6690	-5.2017	-11.0320	-9.5956	-7.48650	-6.1433
Ljung-Box Test (p-value)	4.7510 (0.4669)	10.7790 (0.0340)	1.5692 (0.9497)	1.6363 (0.9437)	1.9867 (0.9066)	5.3038 (0.3862)
ARCH LM Test (p-value)	3.0930 (0.4961)	1.2179 (0.8757)	1.7585 (0.7683)	1.0905 (0.8984)	0.5556 (0.9731)	3.0120 (0.5114)

Note: *** significant at 0.01, ** significant at 0.05, * significant at 0.1.

3-2- ARMA-GARCH-EVT estimation

Accurately capturing the tail behavior of asset returns is crucial for quantifying capital requirements under stress scenarios. To achieve this, an ARMA-GARCH-EVT model was employed, focusing on the left tail due to the generally observed negative skewness. Following a Gaussian ARMA-GARCH specification (as determined in Section 3.1), the POT method was employed for EVT implementation. Selecting an appropriate threshold is critical in POT, as it directly influences the number of exceedances used to estimate the tail distribution. A low threshold may include observations not truly representative of extreme events, biasing the tail estimation. On the other hand, a high threshold may result in too few exceedances, leading to high variance in tail estimates [70].

While various studies such as [41, 71-75] have employed different thresholds for POT, often ranging from the 90th to 95th percentiles, our study specifically focuses on a 95th percentile threshold. This choice stems from our objective of capturing extreme tail risk relevant for solvency considerations, which typically involve higher confidence levels than those used for general risk management. This yielded a consistent number of approximately 143 exceedances across all assets, despite potentially different threshold values (u), indicating the model's effectiveness in capturing tail events across different asset classes. This consistency further supports the suitability of the chosen threshold for our analysis. As shown in Table 3, the estimated shape parameter (ξ) for all assets is close to zero, suggesting an exponential distribution for the tails. This finding is supported by the Kolmogorov-Smirnov test (KS Test), where p-values greater than 0.05 indicate that the null hypothesis of an exponential distribution cannot be rejected. These results collectively validate the model's applicability in assessing investment risk. Importantly, our findings suggest that models assuming a normal distribution may significantly underestimate the capital required by Thai insurers to cover potential investment losses under stress scenarios. This underestimation could jeopardize insurer solvency and, ultimately, policyholder protection.

Table 3. Parameter estimation results of the ARMA-GARCH-EVT model

	u	N_u	AIC	β (Lower, Upper)	Standard Error of β	ξ (Lower, Upper)	Standard Error of α	Distribution	KS Test
SET Index ARMA(3,3)-ARCH(1,1)	1.7790	143	183.952	0.6546 (0.4971, 0.8122)	0.0803	0.0598 (-0.1166, 0.2362)	0.0900	exponential	0.711
Brent Crude Oil ARMA(3,2)-ARCH(1,1)	1.6891	143	189.675	0.6169 (0.4683, 0.7655)	0.0758	0.1392 (-0.0387, 0.3172)	0.0908	exponential	0.773
GOV2 3-7 TTM ARMA(1,1)-ARCH(1,1)	1.5115	143	232.340	0.7274 (0.5256, 0.9293)	0.1029	0.1235 (-0.1012, 0.3483)	0.1146	exponential	0.418
GOV3 7-10 TTM ARMA(2,1)-ARCH(1,1)	1.5868	143	227.4788	0.7390 (0.5639, 0.9142)	0.0893	0.0907 (-0.0810, 0.2624)	0.0876	exponential	0.954
JPY/THB ARMA(2,0)-ARCH(1,1)	1.5493	143	149.882	0.5622 (0.4248, 0.6996)	0.0701	0.0928 (-0.0890, 0.2748)	0.0928	exponential	0.929
Property Index ARMA(1,1)-ARCH(1,1)	1.7314	143	190.356	0.6523 (0.4882, 0.8163)	0.0837	0.0858 (-0.1054, 0.2771)	0.0976	exponential	0.486

3-3- DCC-GARCH Estimation

Table 4 presents the log-likelihood, AIC, and optimal asset allocations for minimizing portfolio VaR for both life and non-life insurers based on the DCC-GARCH estimated model. While a direct comparison is not possible without alternative model specifications, the high log-likelihood values and relatively low AIC values suggest that the DCC-GARCH model provides a reasonable representation of the data for both life and non-life insurer portfolios. As expected, the life insurer adopts a more conservative approach, allocating a significant portion of their portfolio (57%) to long-duration government bonds, with only 13% in stocks and minimal allocations to crude oil (5%) and property (1%). This strategy aligns with the goal of matching asset duration to their longer-term liability profiles. In contrast, the non-life insurer, facing shorter-term liabilities, adopts a more diversified approach. They allocate 30% to each of the riskier asset classes: stocks, exchange rates, and property. This allocation reflects a higher risk tolerance and seeks to maximize returns within acceptable risk limits. They allocate a smaller proportion of their portfolio (5%) to shorter-duration bonds.

Table 4. Estimation of the DCC-GARCH model

Portfolio for Life Insurer	Investment Assets				
	SET Index	**Brent Crude Oil**	**GOV3 7-10 TTM**	**JPY/THB**	**Property Index**
Weight	0.13	0.05	0.57	0.15	0.01
Log-likelihood	51779.82				
AIC	-36.263				
Portfolio for Non-Life Insurer	**Investment Assets**				
	SET Index	**Brent Crude Oil**	**GOV2 3-7 TTM**	**JPY/THB**	**Property Index**
Weight	0.3	0.05	0.05	0.3	0.3
Log-likelihood	53822.49				
AIC	-37.696				

Table 5 presents the unconditional correlation matrices of asset returns for portfolios mimicking the investment durations of life and non-life insurers, respectively. As anticipated, government bonds and the JPY/THB exchange rate exhibit negative correlations with the SET index and Brent crude oil, suggesting potential diversification benefits. While most correlations remain below 0.80, indicating low co-movement, a notable exception exists between property and stock, with a high correlation of 0.851. The high correlation between property and stocks underscores the limitation of static correlation measures. Employing the DCC-GARCH model allows for capturing the time-varying nature of these relationships, leading to more accurate VaR estimations, especially when using a rolling window forecasting method. This dynamic approach is crucial for the subsequent VaR analysis, which utilizes a rolling window forecasting method and evaluates model performance using Kupiec and Christoffersen tests, as presented in the next section.

Table 5. Correlation coefficient matrix

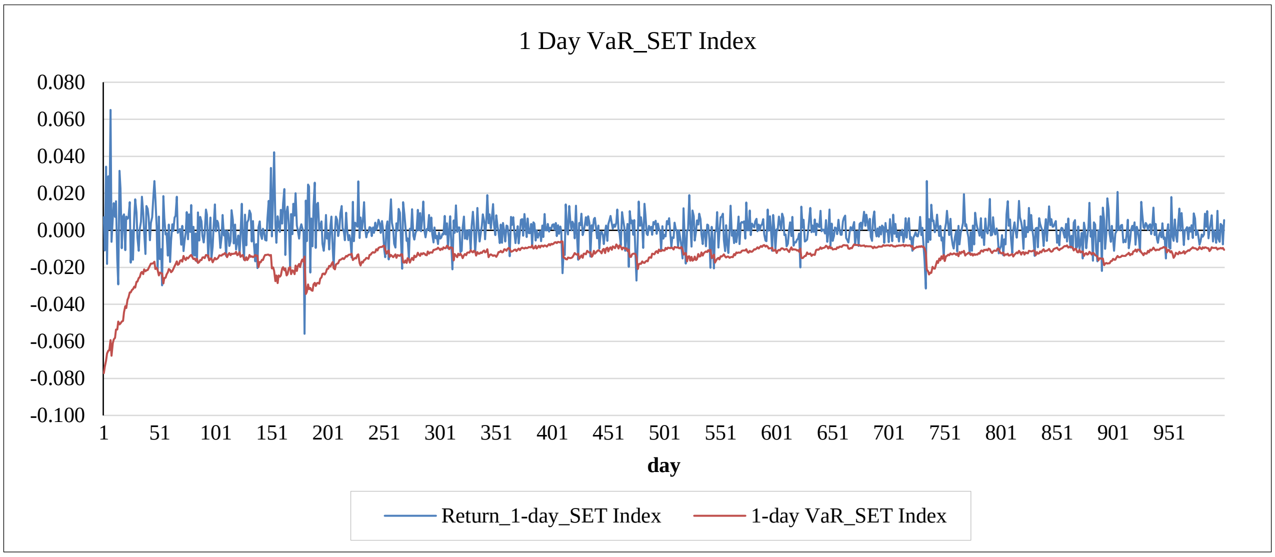
	Investment Portfolio for Life Insurer				
	SET Index	**Brent Crude Oil**	**GOV3 7-10 TTM**	**JPY/THB**	**Property Index**
SET Index	1				
Brent Crude Oil	0.1778	1			
GOV3 7-10 TTM	-0.0056	-0.0504	1		
JPY/THB	-0.2397	-0.1923	0.0369	1	
Property Index	0.8517	0.1172	0.0220	-0.1998	1
	Investment Portfolio for Non-Life Insurer				
	SET Index	**Brent Crude Oil**	**GOV2 3-7 TTM**	**JPY/THB**	**Property Index**
SET Index	1				
Brent Crude Oil	0.1778	1			
GOV2 3-7 TTM	-0.0244	-0.0500	1		
JPY/THB	-0.2397	-0.1923	0.0380	1	
Property Index	0.8517	0.1172	0.0023	-0.1998	1

3-4- Value-at-Risk Backtesting

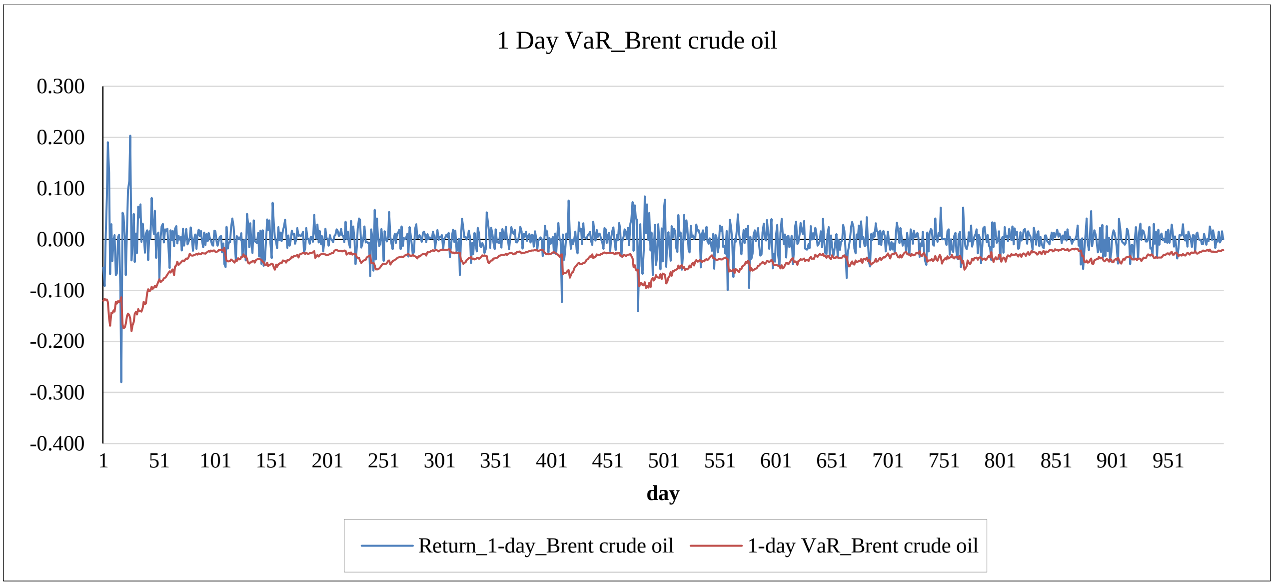
This section evaluates the accuracy of our VaR forecasts using the Kupiec and Christoffersen backtests. A rolling window approach (detailed in Section 2.4) generates 1,000 daily out-of-sample VaR forecasts. The performance of three models is examined across different VaR horizons: 1) ARMA-GARCH: Generates both 1-day-ahead VaR forecasts at a 95% confidence level (aligning with Thai OIC regulations) and 10-day-ahead VaR forecasts at a 99% confidence level (as specified by Basel accords). 2) ARMA-GARCH-EVT: Focuses solely on 1-day-ahead VaR forecasts at a 97.5% confidence level for stress scenario analysis. 3) DCC-GARCH: Similar to ARMA-GARCH, it generates both 1-day-ahead VaR forecasts at a 95% confidence level and 10-day-ahead VaR forecasts at a 99% confidence level.

3-4-1- ARMA-GARCH Model Performance

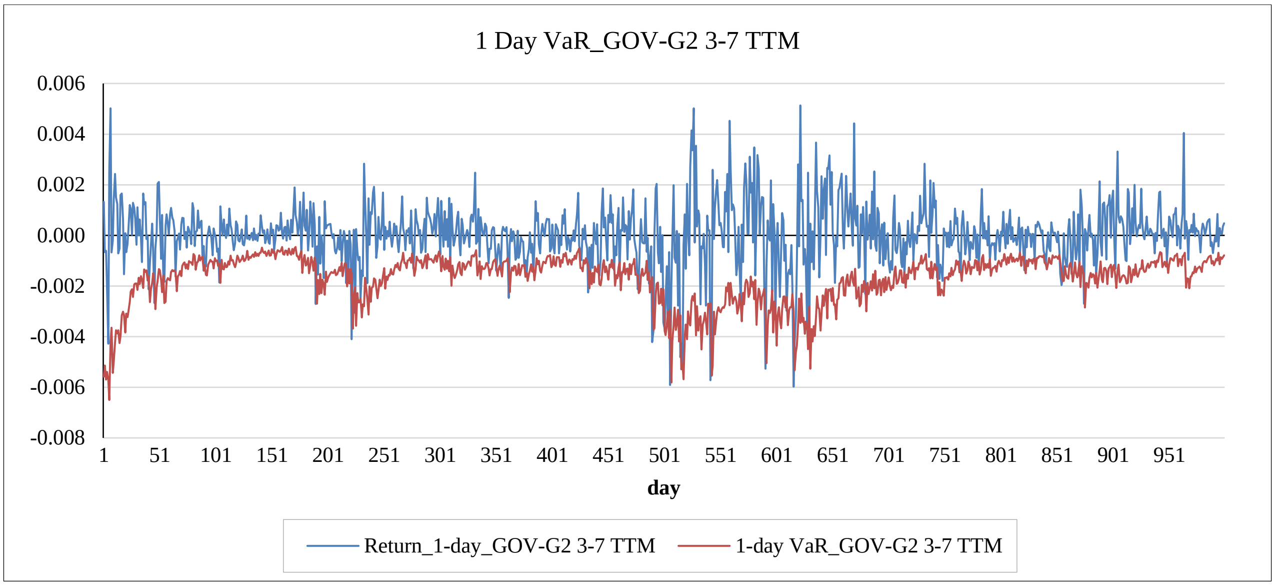
Figures 2 (a-f) and 3 (a-f) and Tables 6 and 7 present the backtesting results for the ARMA-GARCH forecasts. For the 1-day VaR, the model demonstrates robust performance across all assets. Both the Kupiec and Christoffersen tests indicate statistically accurate VaR estimates, suggesting that the ARMA-GARCH model, under the assumption of a normal distribution, is adequate and efficient for one-period 95% VaR forecasts. However, the model's performance for the 10-day VaR reveals potential limitations in capturing tail risk dynamics over longer horizons, as significant deviations from expected exceedances are observed for Brent crude oil, government bond 3-7 TTM, and government bond 7-10 TTM, suggesting the model may not adequately account for the potential clustering of volatility or risk events over longer time horizons for these specific assets. Despite these limitations, the VaR estimates, representing the capital requirements insurers need to hold to protect their solvency, range from 0.17% to 4.23% for a 1-day horizon and from 0.71% to 18.78% for a 10-day horizon of insurers' investments. Notably, the average 10-day-ahead VaR for the SET index, indicating a capital requirement of 6.04%, is slightly higher than those reported for stock indices in developed markets by Degiannakis et al. (2014) [76].



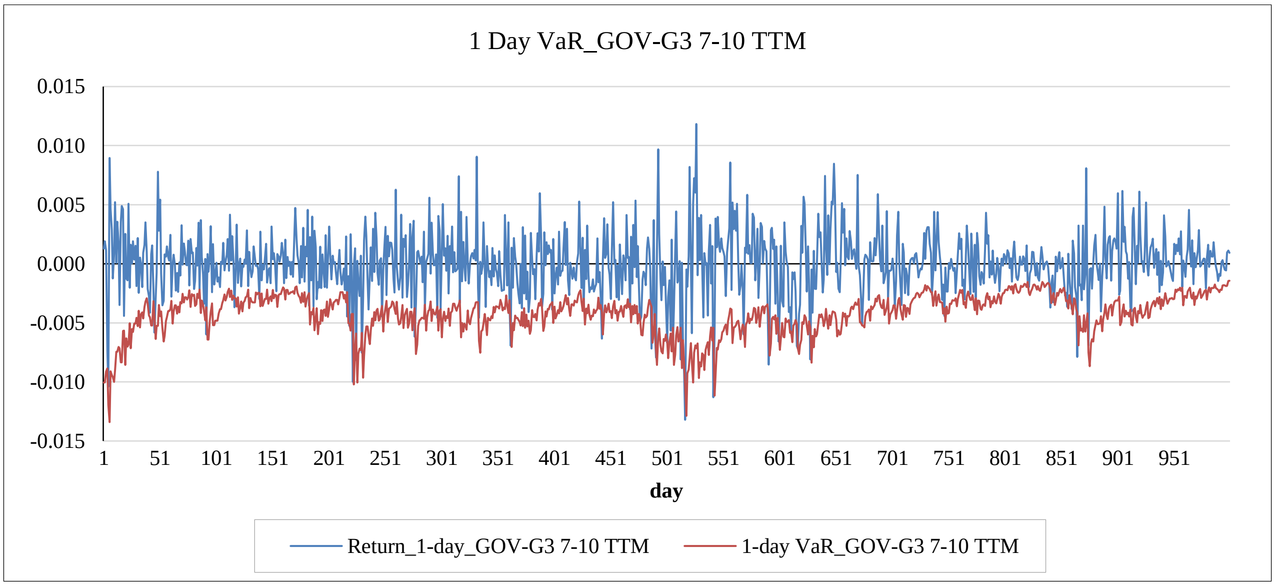
(a) The SET index



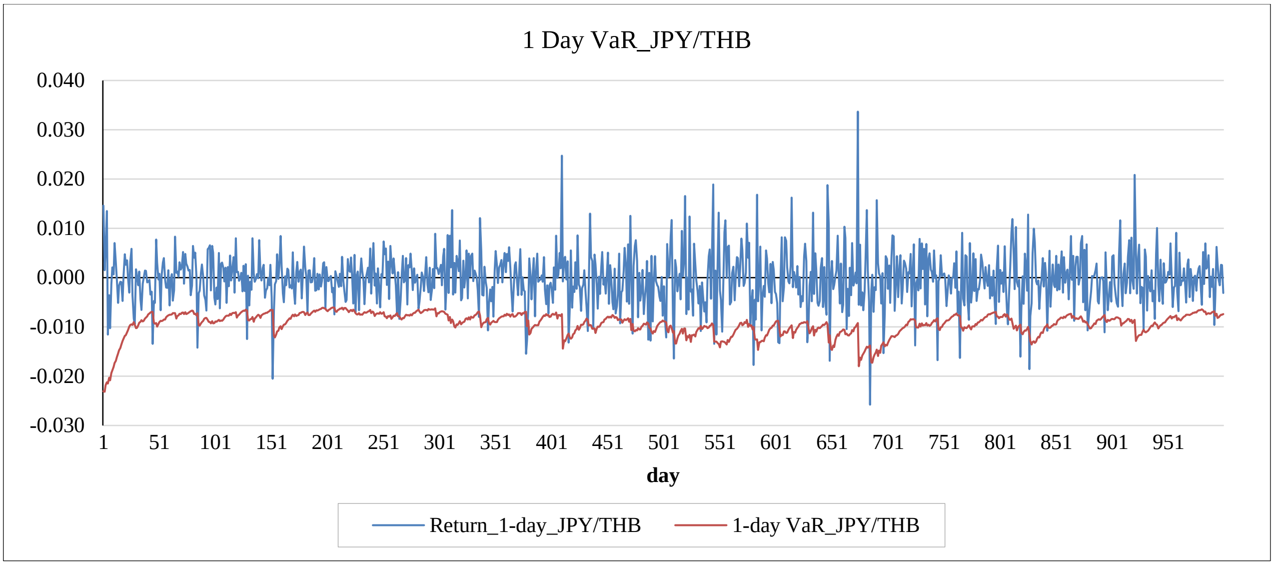
(b) Brent crude oil price



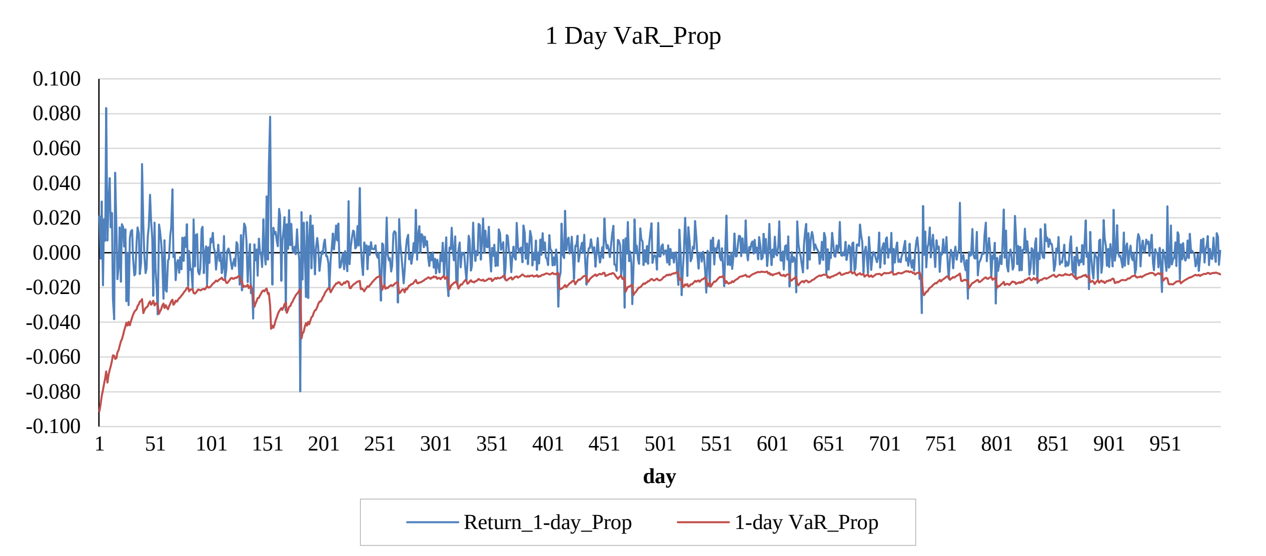
(c) Government bond price (3-7 TTM)



(d) Government bond price (7-10 TTM)

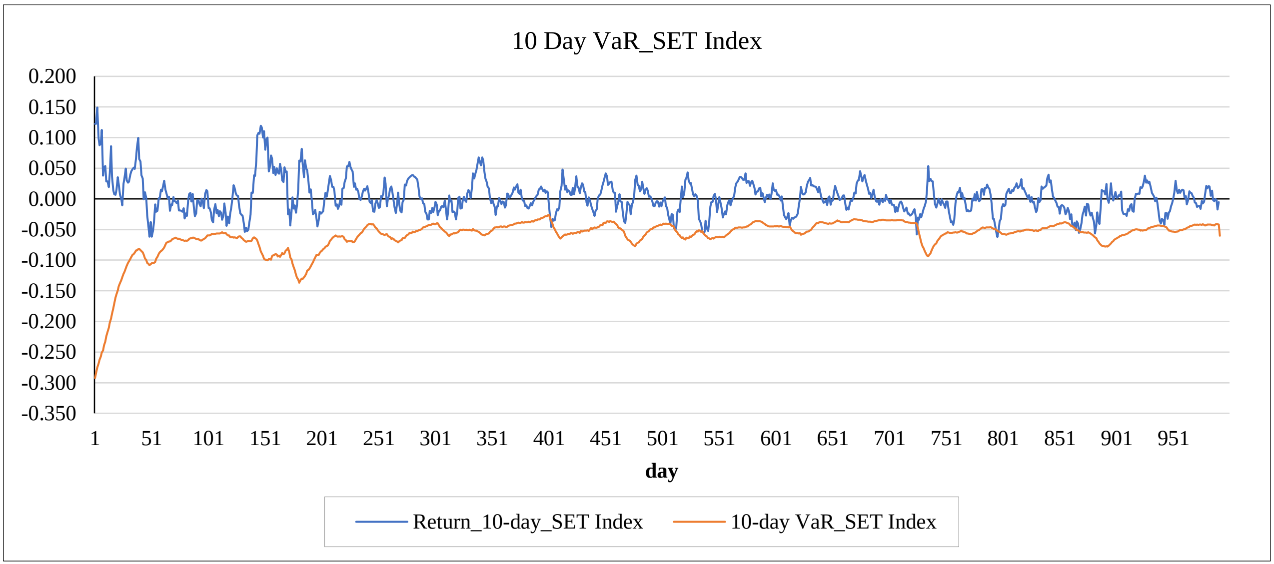


(e) The JPY/THB exchange rate

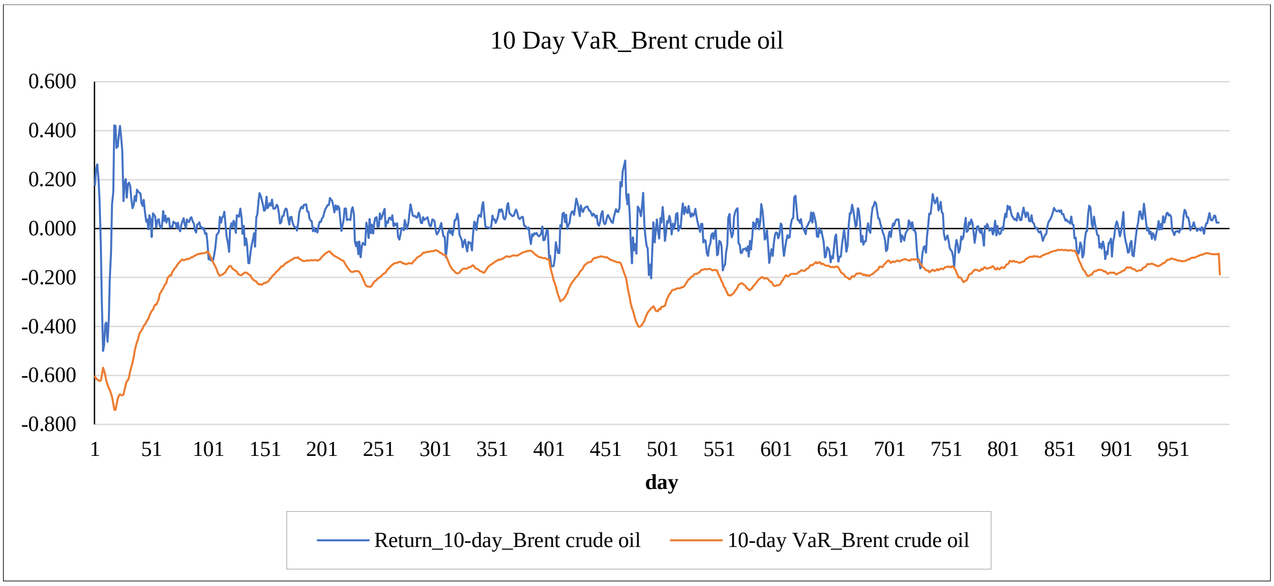


(f) The Property development sector index

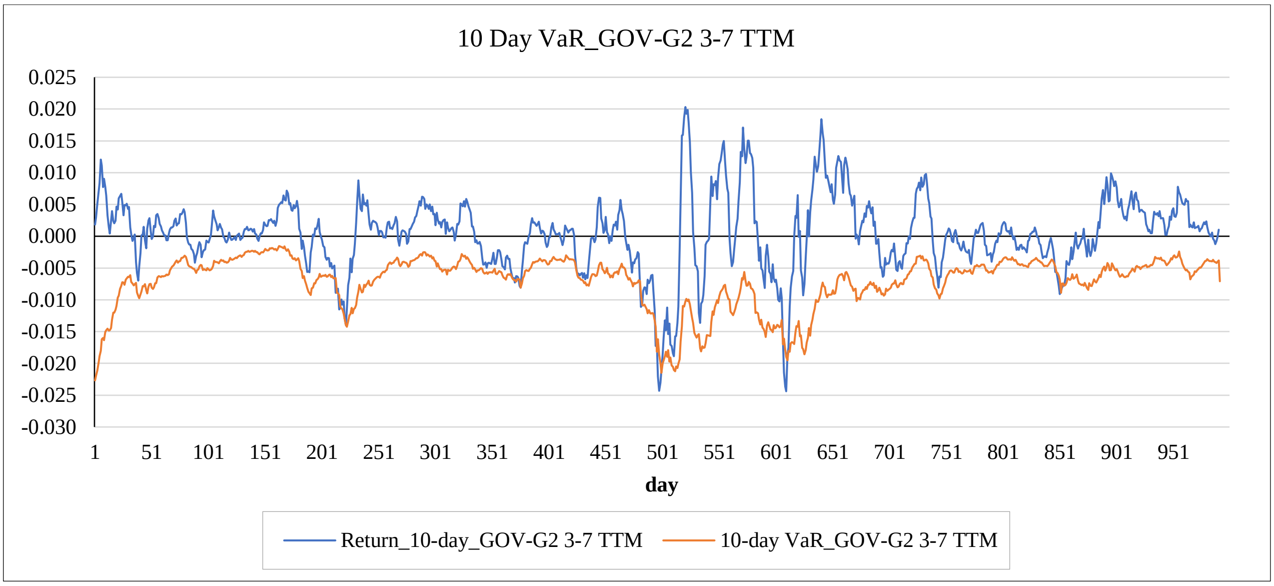
Figure 2. **1-day-ahead VaR forecasting of (a) the SET index, (b) Brent crude oil price, (c) Government bond price (3-7 TTM), (d) Government bond price (7-10 TTM), (e) the JPY/THB exchange rate, and (f) the Property development sector index.**



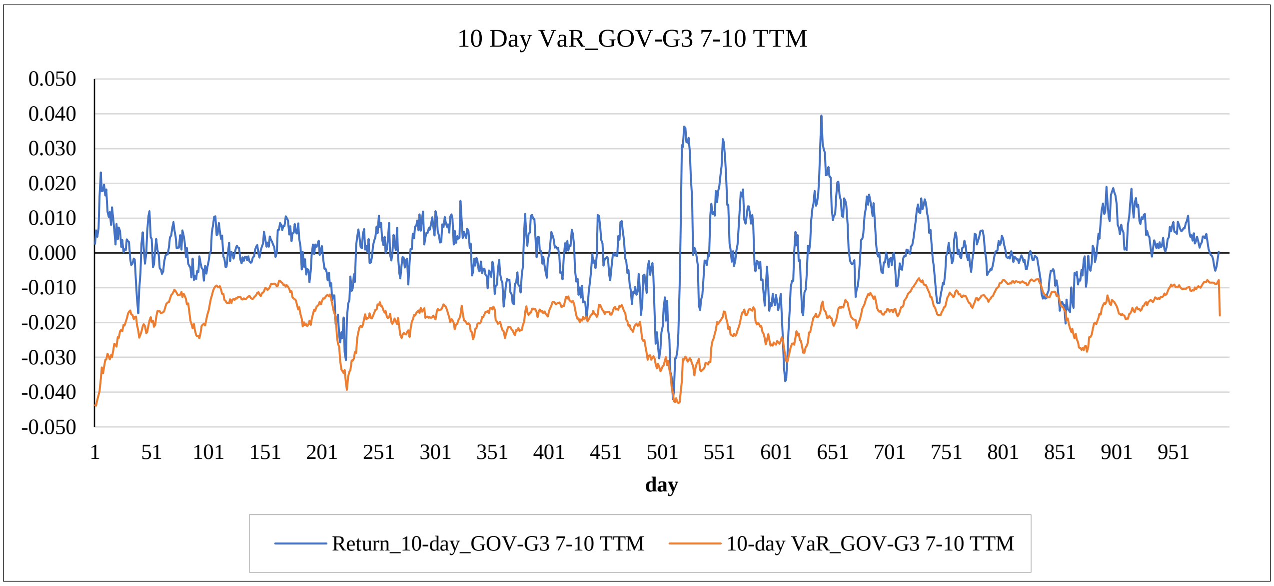
(a) The SET index



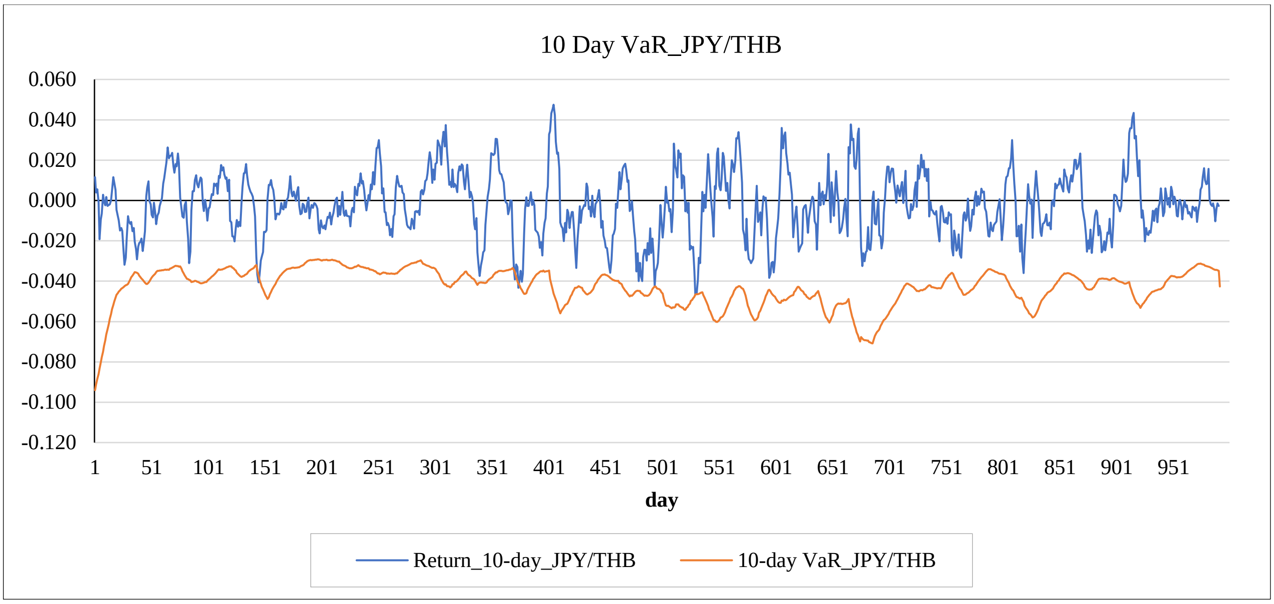
(b) Brent crude oil price



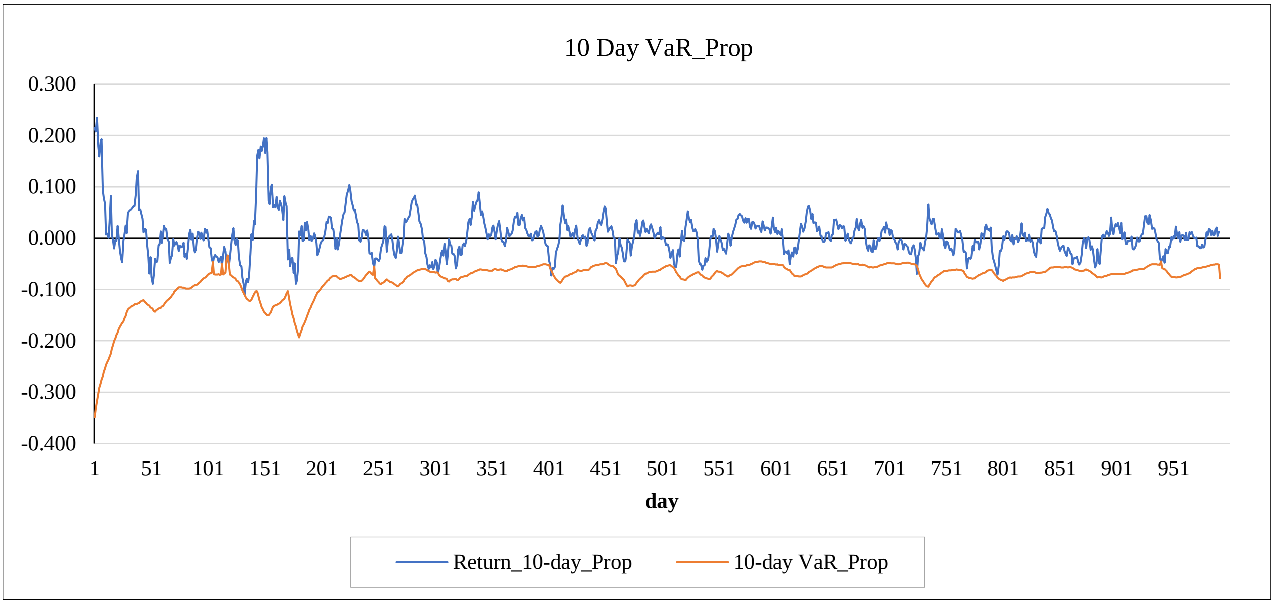
(c) Government bond price (3-7 TTM)



(d) Government bond price (7-10 TTM)



(e) The JPY/THB exchange rate



(f) The Property development sector index

Figure 3. 10-day-ahead VaR forecasting of (a) the SET index, (b) Brent crude oil price, (c) Government bond price (3-7 TTM), (d) Government bond price (7-10 TTM), (e) the JPY/THB exchange rate, and (f) the Property development sector index.

Table 6. 1-day-ahead VaR backtesting: ARMA-GARCH performance

Asset Model	Average 1-day-ahead VaR at 95%	Observed Exceedance Rate	Kupiec Test	Kupiec's p-value	Christoffersen Test	Christoffersen's p-value
SET Index ARMA(3,3)-GARCH(1,1)	-1.42%	5.5%	0.510	0.475	0.842	0.656
Brent Crude Oil ARMA(3,2)-GARCH(1,1)	-4.23%	5.5%	0.510	0.475	0.521	0.770
GOV2 3-7 TTM ARMA(1,1)-GARCH(1,1)	-0.17%	5.6%	0.731	0.392	1.269	0.530
GOV3 7-10 TTM ARMA(2,1)-GARCH(1,1)	-0.42%	4.8%	0.085	0.770	1.229	0.541
JPY/THB ARMA(2,0)-GARCH(1,1)	-0.93%	4.3%	1.081	0.298	1.079	0.583
Property Index ARMA(1,1)-GARCH(1,1)	-1.84%	3.8%	3.294	0.069	3.476	0.176

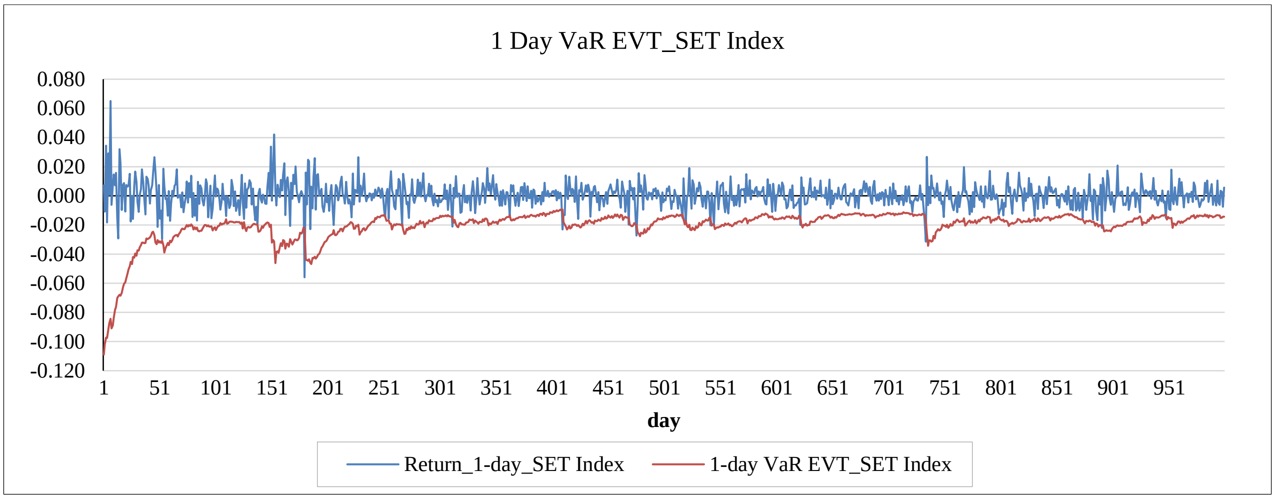
Table 7. 10-day-ahead VaR backtesting: ARMA-GARCH performance

Asset Model	Average 10-day-ahead VaR at 99%	Observed Exceedance Rate	Kupiec Test	Kupiec's p-value	Christoffersen Test	Christoffersen's p-value
SET Index ARMA(3,3)-GARCH(1,1)	-6.04%	0.9%	0.087	0.768	3.456	0.178
Brent Crude Oil ARMA(3,2)-GARCH(1,1)	-18.78%	0.7%	0.962	0.327	22.664	0.000***
GOV2 3-7 TTM ARMA(1,1)-GARCH(1,1)	-0.71%	3.2%	8.012	0.004***	55.755	0.000***
GOV3 7-10 TTM ARMA(2,1)-GARCH(1,1)	-1.80%	1.5%	2.282	0.131	53.970	0.000***
JPY/THB ARMA(2,0)-GARCH(1,1)	-4.27%	0.5%	3.003	0.083	8.824	0.012
Property Index ARMA(1,1)-GARCH(1,1)	-7.87%	1.0%	0.001	0.972	8.933	0.012

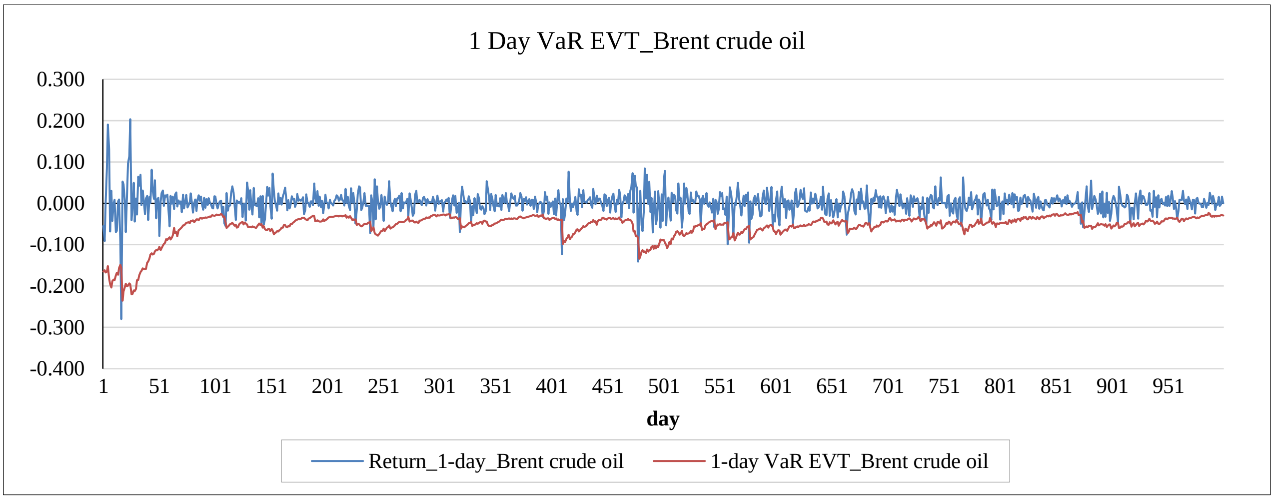
Note: *** significant at 0.01

3-4-2- ARMA-GARCH-EVT Model Performance

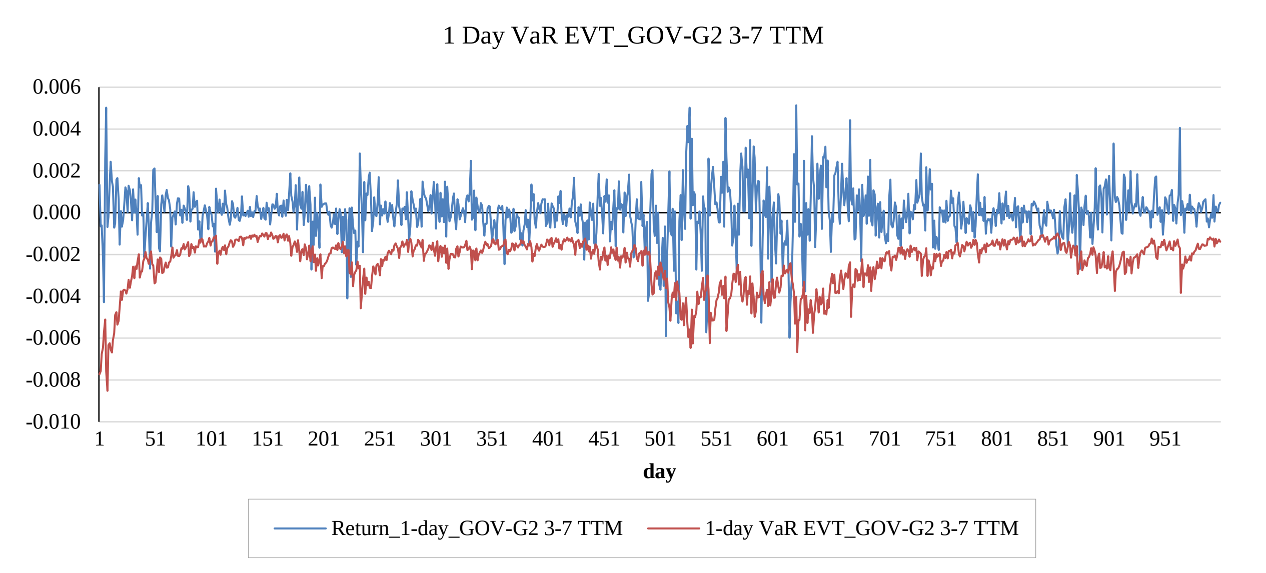
To better capture tail risk, this study employs the ARMA-GARCH-EVT model, incorporating EVT, to generate 1-day-ahead VaR forecasts at a 97.5% confidence level. Figures 4 (a-f) and Table 8 present the backtesting results, which, similar to the ARMA-GARCH model, demonstrate robust performance of the ARMA-GARCH-EVT model in 1-day-ahead VaR forecasting, with the exception of government bond 3-7 TTM. While both the Kupiec and Christoffersen tests confirm the accuracy of most VaR estimates, the Christoffersen test rejects the accuracy for the Government bond 3-7 TTM. The VaR estimates derived from the ARMA-GARCH-EVT model, ranging from 0.23% to 5.58% of insurers' investments for a 1-day horizon, are crucial as they represent the capital requirements insurers would need to hold under a stress scenario to maintain solvency. Of note, the average 1-day-ahead VaR for the SET index, indicating a capital requirement of 2.03%, is lower than the 2.60% to 3.99% range reported for stock indices in developed markets, which are calculated at a 99% confidence level, by Echaust & Just (2020) [77]. This difference highlights potential variations in capital adequacy standards across markets.



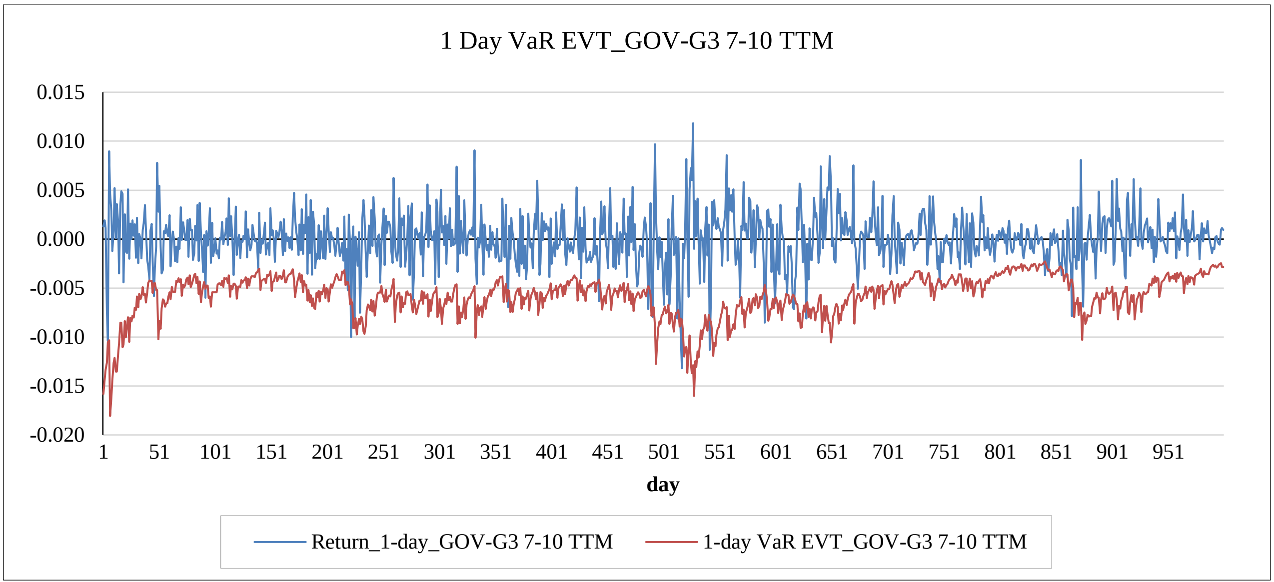
(a) The SET index



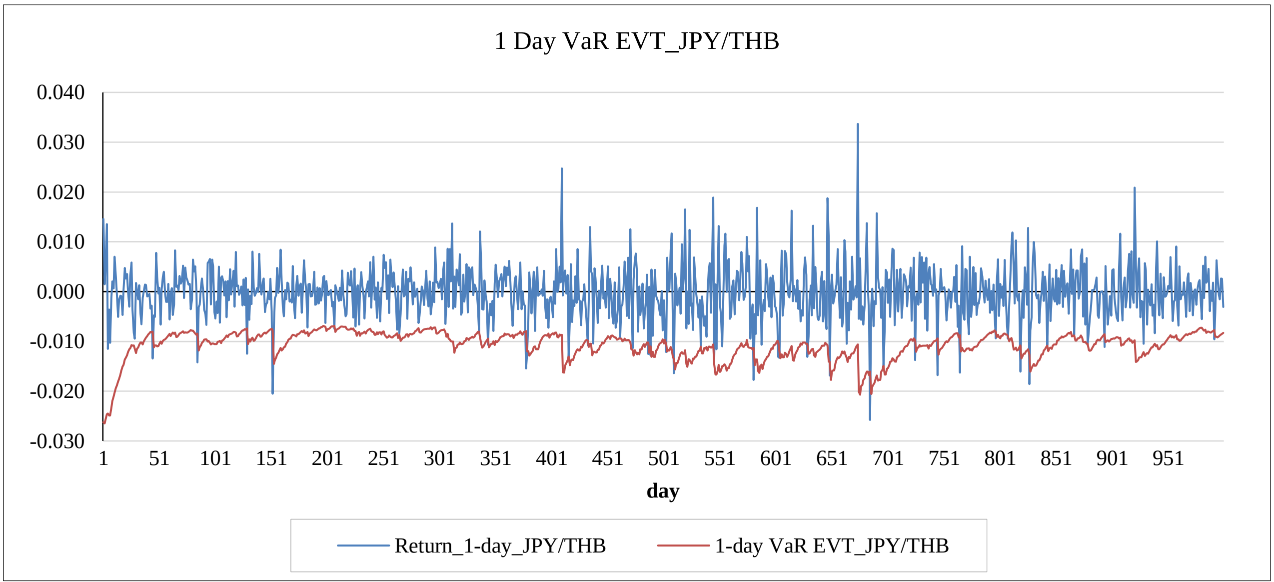
(b) Brent crude oil price



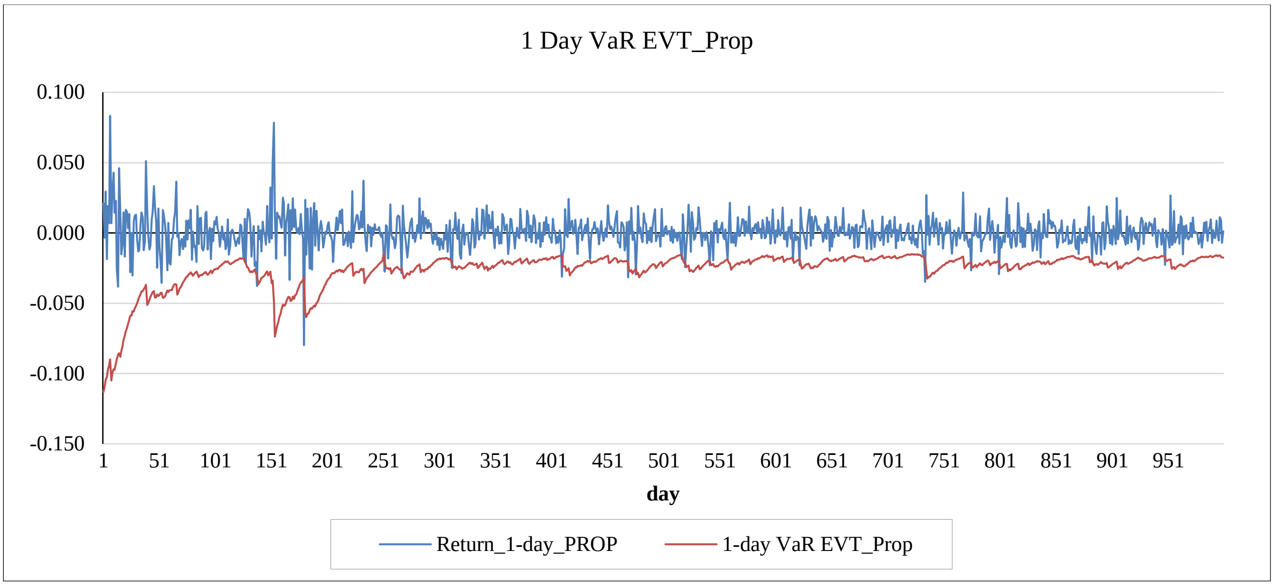
(c) Government bond price (3-7 TTM)



(d) Government bond price (7-10 TTM)



(e) The JPY/THB exchange rate



(f) The Property development sector index

Figure 4. 1-day-ahead EVT VaR forecasting of (a) the SET index, (b) Brent crude oil price, (c) Government bond price (3-7 TTM), (d) Government bond price (7-10 TTM), (e) the JPY/THB exchange rate, and (f) the Property development sector index

Table 8. 1-day-ahead VaR backtesting: ARMA-GARCH-EVT performance

Asset Model	Average 1-day-ahead VaR at 95%	Observed Exceedance Rate	Kupiec Test	Kupiec's p-value	Christoffersen Test	Christoffersen's p-value
SET Index ARMA(3,3)-GARCH(1,1)	-2.03%	1.8%	2.224	0.136	3.163	0.205
Brent Crude Oil ARMA(3,2)-GARCH(1,1)	-5.58%	2.6%	0.041	0.839	0.081	0.961
GOV2 3-7 TTM ARMA(1,1)-GARCH(1,1)	-0.23%	3.4%	2.992	0.084	33.799	0.000***
GOV3 7-10 TTM ARMA(2,1)-GARCH(1,1)	-0.58%	2.4%	0.042	0.838	2.347	0.309
JPY/THB ARMA(2,0)-GARCH(1,1)	-1.07%	3.1%	1.374	0.241	2.312	0.315
Property Index ARMA(1,1)-GARCH(1,1)	-2.59%	2.2%	0.385	0.535	0.822	0.663

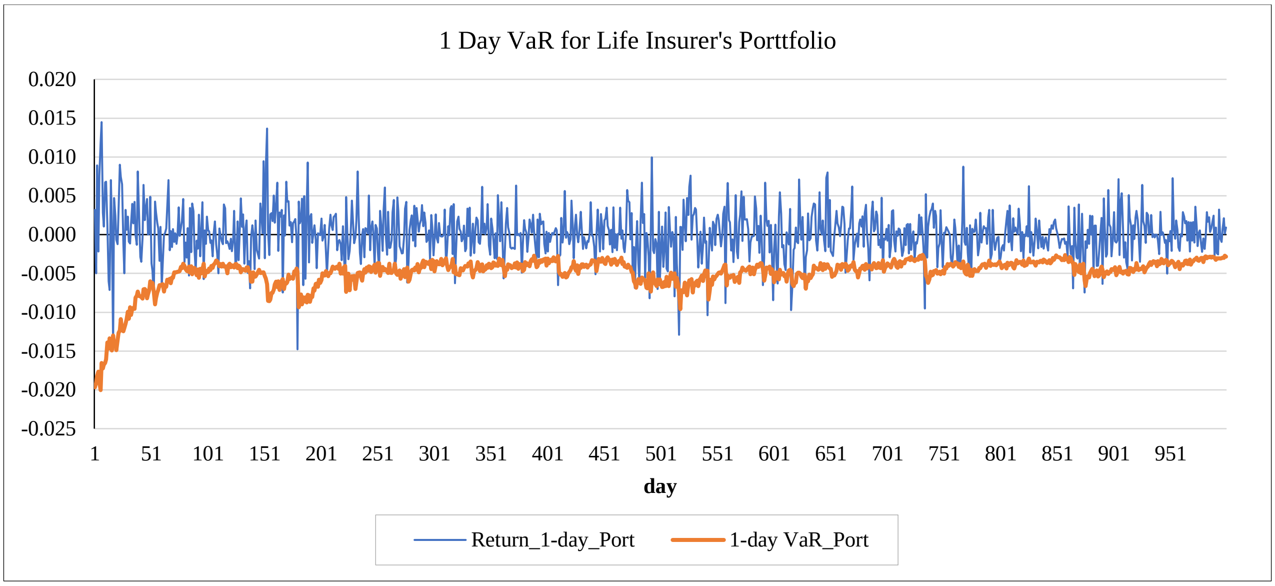
Note: *** significant at 0.025.

3-4-3- DCC-GARCH Model Performance

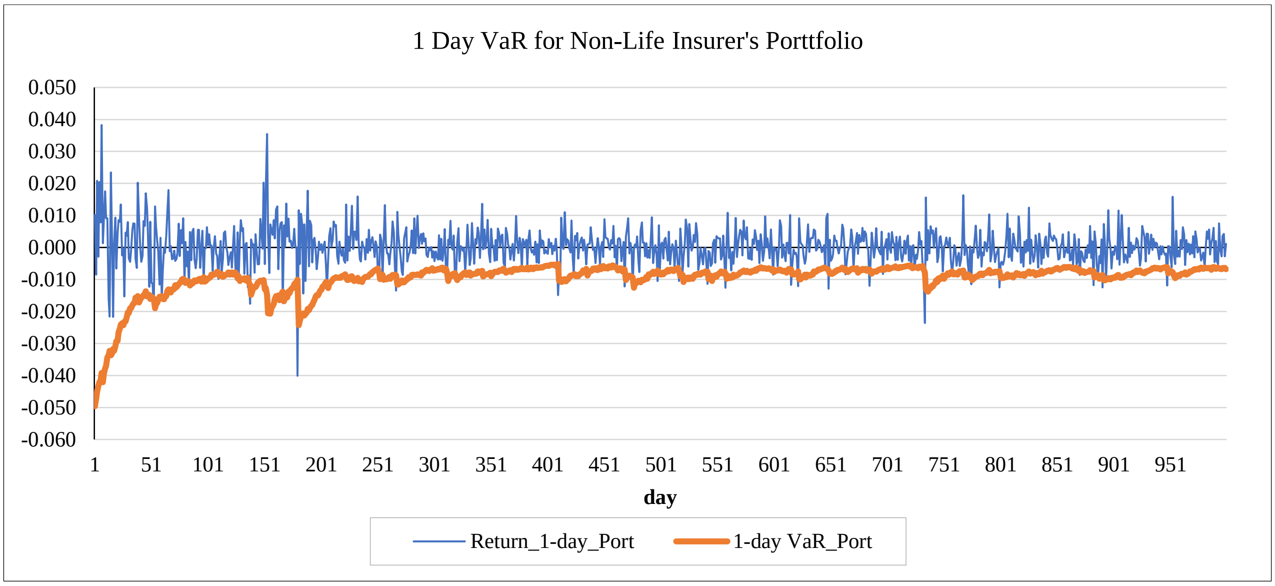
This study examined VaR forecasts using ARMA-GARCH, ARMA-GARCH-EVT, and ultimately the DCC-GARCH model, which explicitly accounts for time-varying correlations between assets – a crucial factor for assessing diversification benefits. This feature is particularly relevant for assessing the diversification benefits for life and non-life insurers, which employ distinct investment strategies aimed at matching asset duration to liability profiles and minimizing portfolio VaR. Accurately capturing these benefits is crucial for optimizing asset allocation and ensuring sufficient capital reserves. As highlighted in Table 5, while static correlations suggest potential diversification benefits, such as the negative correlations between government bonds and the SET index and between the JPY/THB exchange rate and Brent crude oil, these relationships can fluctuate significantly over time. The DCC-GARCH model captures this dynamic, leading to more accurate VaR estimations. Figures 5 (a-b) and 6 (a-b) and Tables 9 and 10 present the backtesting results, confirming the statistical accuracy of the DCC-GARCH VaR forecasts across both 1-day and 10-day horizons.

The findings reveal notable differences in capital requirements between life and non-life insurers, even when both seek to match portfolio duration to their liabilities. The analysis indicates 1-day and 10-day VaR portfolio estimates of 0.5% and 2.09% for life insurers, and 0.96% and 4.12% for non-life insurers, respectively. This difference underscores the impact of varying asset allocation strategies driven by the nature of their liabilities, with life insurers typically holding more long-duration assets like government bonds. This aligns with [78], which highlighted factors influencing risk-taking and performance differences between life and non-life insurers. Importantly, the DCC-GARCH model demonstrates the benefits of diversification in reducing overall portfolio risk. Compared to a hypothetical portfolio constructed using constant correlations from Table 5, VaR estimates were reduced by an average of 48.37% (1-day) and

48.98% (10-day) for life insurers, and 34.70% (1-day) and 35.88% (10-day) for non-life insurers. These substantial risk reductions highlight the limitations of static correlation measures and underscore the importance of considering dynamic correlations, especially in light of findings from [79, 80], which demonstrated this importance in the context of international investments. This study builds upon this understanding by demonstrating these benefits within a domestic market, specifically highlighting the differences between life and non-life insurers.



(a) Life insurer investment portfolio



(b) Non-life insurer investment portfolio

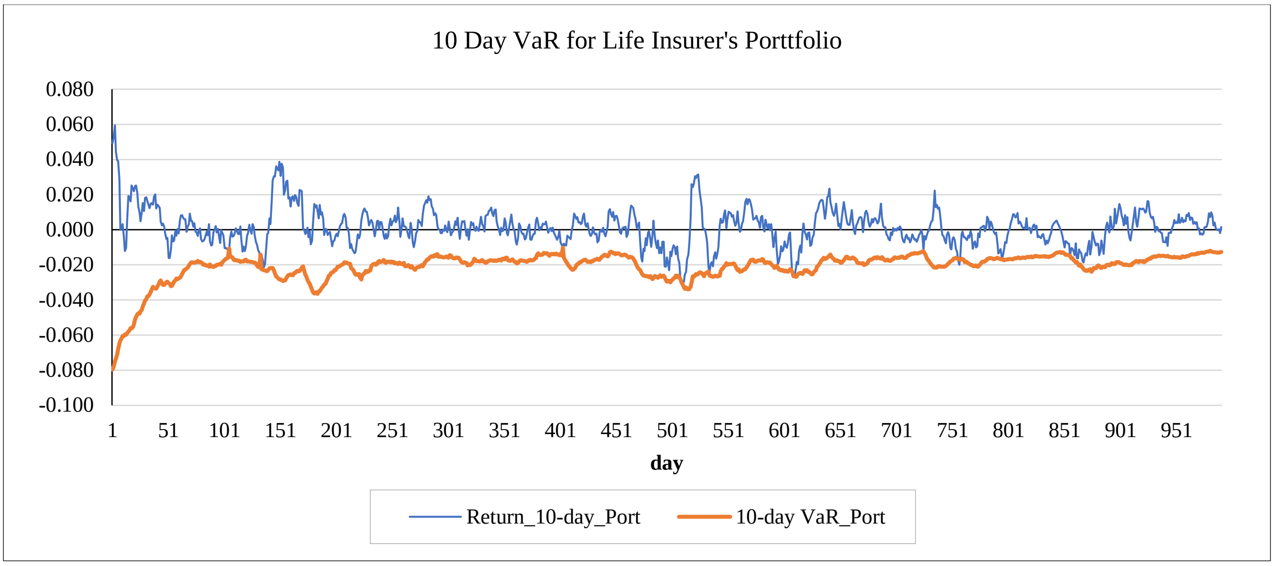
Figure 5. 1-day-ahead VaR forecasting of (a) life insurer investment portfolio and (b) non-life insurer investment portfolio

Table 9. 1-day-ahead VaR backtesting: DCC-GARCH performance

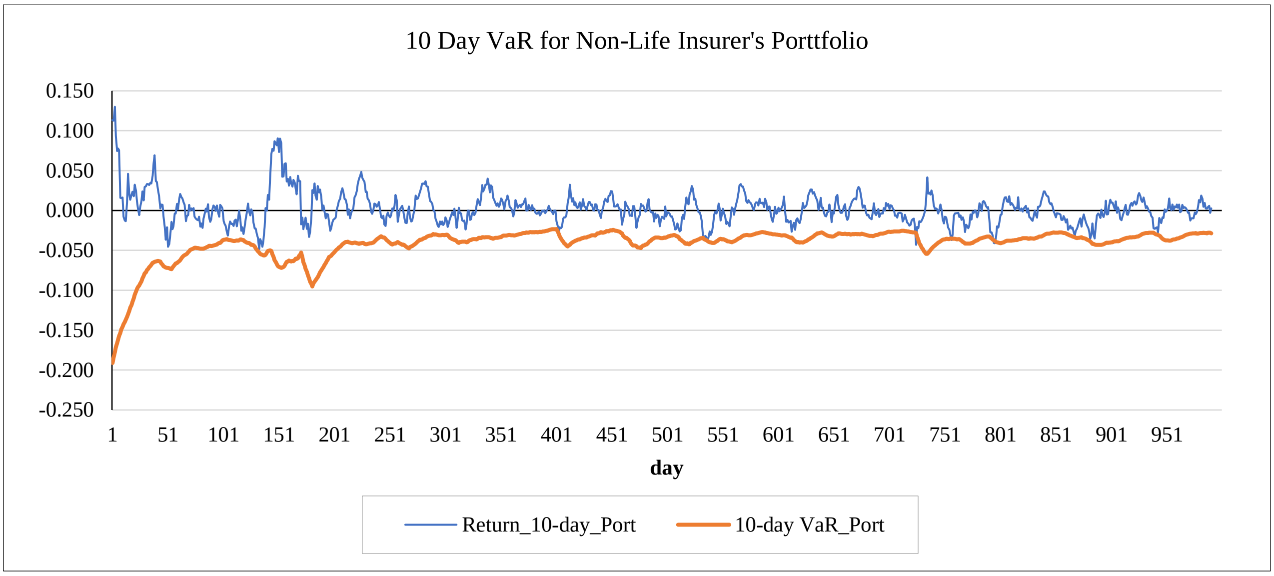
Portfolio Model	Average 1-day-ahead VaR at 95%	Observed Exceedance Rate	Kupiec Test	Kupiec's p-value	Christoffersen Test	Christoffersen's p-value
Portfolio for Life Insurer	-0.50%	5.3%	0.١٨٦	0.666	0.480	0.787
Portfolio for Non-Life Insurer	-0.96%	5.6%	0.٧٣١	0.393	3.120	0.210

Table 10. 10-day-ahead VaR backtesting: DCC-GARCH performance

Portfolio Model	Average 10-day-ahead VaR at 99%	Observed Exceedance Rate	Kupiec Test	Kupiec's p-value	Christoffersen Test	Christoffersen's p-value
Portfolio for Life Insurer	-2.09%	0.7%	0.962	0.327	5.340	0.069
Portfolio for Non-Life Insurer	-4.12%	0.6%	1.814	0.178	6.838	0.033



(a) Life insurer investment portfolio



(b) Non-life insurer investment portfolio

Figure 6. 10-day-ahead VaR forecasting of (a) life insurer investment portfolio and (b) non-life insurer investment portfolio

4- Conclusion

This research provides a robust framework for assessing and forecasting capital requirements for life and non-life insurers to effectively navigate evolving regulations and market uncertainties. By employing ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH models, this study captured both typical and tail-risk dynamics, providing a nuanced understanding of insurers' capital adequacy needs. Rigorous backtesting procedures confirm the overall effectiveness of these models in accurately forecasting VaR. Notably, the integration of EVT significantly enhanced stress testing capabilities, proving particularly valuable in volatile markets. However, a small subset of 10-day VaR models revealed limitations, potentially due to structural breaks not fully captured in the model. This highlights the need for insurers to consider potential regime changes, such as geopolitical events impacting oil or central bank announcements affecting bond markets, when making decisions based on longer-horizon VaR forecasts.

Furthermore, the findings underscore the importance of dynamic asset correlations. The DCC-GARCH model demonstrated that diversification strategies, tailored to the specific risk exposures of life and non-life insurers, can significantly reduce capital requirements. This emphasis on dynamic correlations highlights the need for insurers to incorporate time-varying factors into their risk management practices, especially during periods of potential financial disruption. The insights derived from this novel study offer a strong foundation for insurance firms to enhance their risk assessment and capital allocation strategies. By adopting the robust methodological approach demonstrated here, insurers can gain a nuanced understanding of capital requirements and the benefits of dynamic diversification, empowering them to make more informed decisions.

Building upon this focus on ARMA-GARCH, ARMA-GARCH-EVT, and DCC-GARCH models under normal distribution assumptions, future research could explore alternative distributions, such as Student's t-distribution or skewed distributions, and explore the application of other GARCH family models, such as EGARCH or TGARCH, to potentially further refine VaR estimation and offer additional insights into volatility dynamics. Additionally, exploring the GARCH-M model, which incorporates volatility as a factor directly influencing returns, could be particularly relevant for insurers given the potential impact of market volatility on investment strategies and overall portfolio returns. These avenues of exploration will contribute to the ongoing development of resilient and responsive insurance markets.

5- Declarations

5-1- Author Contributions

Conceptualization, T.C. and P.G.; methodology, P.G. and T.C.; software, P.G. and T.C.; validation, P.G. and T.C.; formal analysis, P.G. and T.C.; investigation, T.C. and P.G.; resources, T.C. and P.G.; data curation, P.G. and T.C.; writing—original draft preparation, T.C.; writing—review and editing, T.C. and P.G.; visualization, P.G. and T.C.; corresponding author, P.G.; project administration, T.C.; funding acquisition, T.C. All authors have read and agreed to the published version of the manuscript.

5-2- Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5-3- Funding

This research was supported by the Chulalongkorn Business School, Chulalongkorn University.

5-4- Institutional Review Board Statement

Not applicable.

5-5- Informed Consent Statement

Not applicable.

5-6- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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