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Modelling School Zone Border as Rich Modelling Problem for Secondary School Students

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Abstract

This article explores the potential of using the school zoning problem in Indonesia as a vehicle for teaching mathematical modeling to secondary school students. This problem is highly suitable for students as a modeling challenge because it is (i) contextual, (ii) rich, (iii) challenging, and (iv) within students' Zone of Proximal Development (ZPD). School zoning involves a concept called Voronoi, essentially a partitioning problem. For simpler or special-case problems, these partitions can be created using concepts already taught in secondary schools, such as perpendicular bisectors and radical axes. However, for more complex problems with multiple sites, an algorithm is required, which involves advanced mathematical concepts beyond the typical secondary curriculum. Yet, with the rise of visual programming languages like Scratch, Snap!, StarLogo, and TurboWarp, it becomes possible to tackle these partitioning challenges using coding and only basic mathematical principles. This approach not only enhances students' understanding of foundational mathematical concepts but also fosters the integration of computational thinking and coding within mathematics. In summary, the school zoning problem serves as an ideal topic for mathematical modeling for secondary school students, promoting the integration of mathematical concepts, computational thinking, and coding skills.

Keywords:

Modelling;

Basic Mathematical Concepts;

Visual Programming Languages;

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1- Introduction

The primary issue facing mathematics education today is low student engagement, which consequently leads to low student achievement. Numerous international surveys highlight the poor quality of mathematics education in our country, one of which is the well-known PISA survey [1]. In the most recent 2018 PISA results, Indonesia ranked 74th in mathematics, placing us as the 6th lowest-ranking country. Our mathematics score was only 379, well below the OECD average of 479. The top five countries in this ranking were dominated by our neighboring nations: China, Singapore, Chinese Taipei, and Japan.

According to the World Bank, Indonesia ranked 87th out of 157 countries on the 2018 Human Capital Index, which measures future productivity potential based on education and health outcomes [2, 3]. Indonesia's Human Capital Index score was only 0.53, meaning that, on average, the next generation of Indonesian workers will be only 53% as productive as they could be under the ideal benchmark of 14 years of learning and full health [4, 5]. The World Bank further noted that, despite significant progress in previous years, most students still do not meet the national learning targets set by our country. Student learning outcomes are low both in absolute terms—below national targets—and relative to neighboring countries [6]. Despite recent improvements in learning, as measured by PISA, it is estimated that it will take Indonesia 50 years to reach the current OECD average score [7, 8].

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Based on these results, it is clear that we need to improve the quality of mathematics education. Enhancing students' capabilities in mathematical modeling is especially important [9, 10]. This need arises not only because PISA survey items primarily assess the abilities of 15-year-old students to apply studied mathematics through modeling, but also due to a broader shift in education from mere understanding to problem-solving and modeling [9, 11, 12]. Unfortunately, mathematics instruction in Indonesia rarely includes modeling; it is mostly focused on calculations and, at best, the application of concepts and formulas [13, 14].

Previous studies define mathematical modeling as a process that uses mathematics to represent, analyze, make predictions, or provide insights into real-world phenomena [15]. Thus, the most essential aspect of mathematical modeling is the connection between the real world and mathematics [16, 17]. This connection is established through (i) the use of mathematics to quantify real-world problems and analyze their behavior, (ii) the use of mathematics to explore and understand the world, and (iii) an iterative problem-solving process to develop deeper insights [18–20].

Problems designed to achieve these goals should be interesting, engaging, and challenging, encouraging students in the problem-solving process without causing frustration. Given the difficulty in developing problems that meet these criteria, we have explored the theoretical possibility of using the concept of Voronoi diagrams for modeling challenges suitable for secondary school students. This problem is closely related to a real issue faced every year in student recruitment in Indonesia. The country uses a zoning system to assign students to schools, with students expected to attend schools within their designated zones. However, disputes frequently arise across Indonesia regarding student placement, as there is no clear or scientifically accepted method to define school zone boundaries.

Based on the requirements for a good problem in mathematical modeling, the school zoning boundary issue is highly promising. We do not intend to teach students about Voronoi diagrams—although they can solve partitioning problems, they require advanced mathematics beyond the scope of secondary school curriculum. Instead, we will use alternative approaches to address this partitioning issue: (i) the concept of perpendicular bisectors, a mathematical topic introduced in junior secondary school, and (ii) Agent-Based Modeling (ABM), which relies on intuitive reasoning, such as assigning students to the nearest school based on proximity.

However, implementing the second approach requires students to develop Computational Thinking skills, including (i) solving the problem mathematically by defining rules for each "agent" (in this case, each student) to follow, and (ii) creating a code to simulate the behavior of each agent according to these rules. Through this process, an emergent pattern may form, representing the boundaries of a given school zone.

This article is organized as follows: (i) Modeling and Agent-Based Modeling (ABM), (ii) Indonesia's School Zoning Regulations, (iii) Voronoi Diagrams and Related Concepts (Perpendicular Bisectors, Circle Power, and ABM using StarLogo or TurboWarp), and (iv) Case Implementation using data from a junior high school in Singaraja.

2- Modelling and Agent-Based Modelling (ABM)

Throughout mathematics education from elementary to high school, the term "model" has been used in various contexts. It sometimes refers to manipulatives, demonstrations, role models, and, at times, conceptual modeling, such as Systems of Linear Equations in Two Variables (SLETV), among others [21, 22]. In this sense, modeling is indeed a valuable tool for teaching and learning mathematics. However, these examples differ from the practice of mathematical modeling, which, especially in workplace and real-life contexts, uses mathematics to answer complex, real-world questions [23, 24].

Most mathematics teachers agree that mathematical modeling should be taught at every educational level. It not only benefits students as they progress through school and enter the workforce, but it also assists them in daily life and as informed citizens [25, 26]. It is essential for students to encounter a wide variety of modeling problems as they advance through the grades, such as determining the average rainfall in their area, identifying the best location for a fire station, assessing fair voting systems, or arranging pictures along a staircase to look level. Through these experiences, students will recognize the relevance of mathematics, especially modeling, in their lives.

Consider the following example: students are asked to determine the price of a book and a pencil based on the following problem. One day, Mira goes to a bookstore, buys 3 books and 2 pencils, and pays Rp10,500. On another day, Billy goes to the same store, buys 2 books and 1 pencil, and pays Rp6,500. The question is: What is the price of each item? It should be noted that this is not a modeling problem; rather, it is a problem of mathematical application. Mathematical application is not the same as modeling. A true mathematical modeling problem should hold intrinsic value or meaning for students. Beyond solving this problem as a class exercise or homework, why would students care about the price of one book and one pencil? This problem lacks intrinsic value for students. Additionally, it is closed-ended: all necessary data is provided, and there is only one correct solution. These characteristics define it as an application problem, rather than a modeling one.

Now, let's explore the following problem, which is a modeling problem. Imagine you want to help your friend choose the right job so that he can earn a good salary to buy gifts for his girlfriend's birthday. The problem could be framed as follows:

> The holidays are approaching, and your best friend Billy would like to make some money to purchase gifts for his girlfriend. He found one job that pays Rp100,000 above the minimum wage. Another job offers to pay half the minimum wage plus a commission of Rp20,000 for each item he sells. Which job is better? To help Billy understand your analysis, include a useful representation to assist him in making the decision (adapted from GAIMME, 2019) [27].

To answer this problem, students will need to put in more work. They might need to look up the minimum wage in the area and consider the "break-even" point—the number of items Billy will need to sell in order to earn a wage comparable to that from the other job. Additionally, they will need to think about whether it is likely that Billy could sell that many items, which probably depends on the type of item and his personality. Researching contexts and making assumptions about these contexts are important components of mathematical modeling. Thus, mathematical modeling requires significant decision-making on the part of the students [28, 29].

Agent-Based Modeling (ABM) is a computer-based modeling approach that has emerged from the field of complex systems and uses simple computational rules as its fundamental modeling elements [30]. In this model, mathematical rules are assigned to each individual, called an agent. The effects of these rules are then investigated over time. For example, a simple population dynamic of fish can be modeled by assigning rules for each fish regarding their movement, feeding, reproduction, and mortality. ABM provides a better epistemological match to our intuitive understanding of parts that constitute systems as distinct individuals or entities [31].

In the field of complex systems, there is another type of modeling known as Systems Dynamics Modeling (SDM). In ABM, mathematical rules are applied at the individual level, whereas in SDM, the rules are applied at the aggregate level. For the fish population example, in SDM, the mathematical rules would be applied to the population as a whole rather than to individual fish. The following examples illustrate how modeling is carried out in both ABM and SDM.

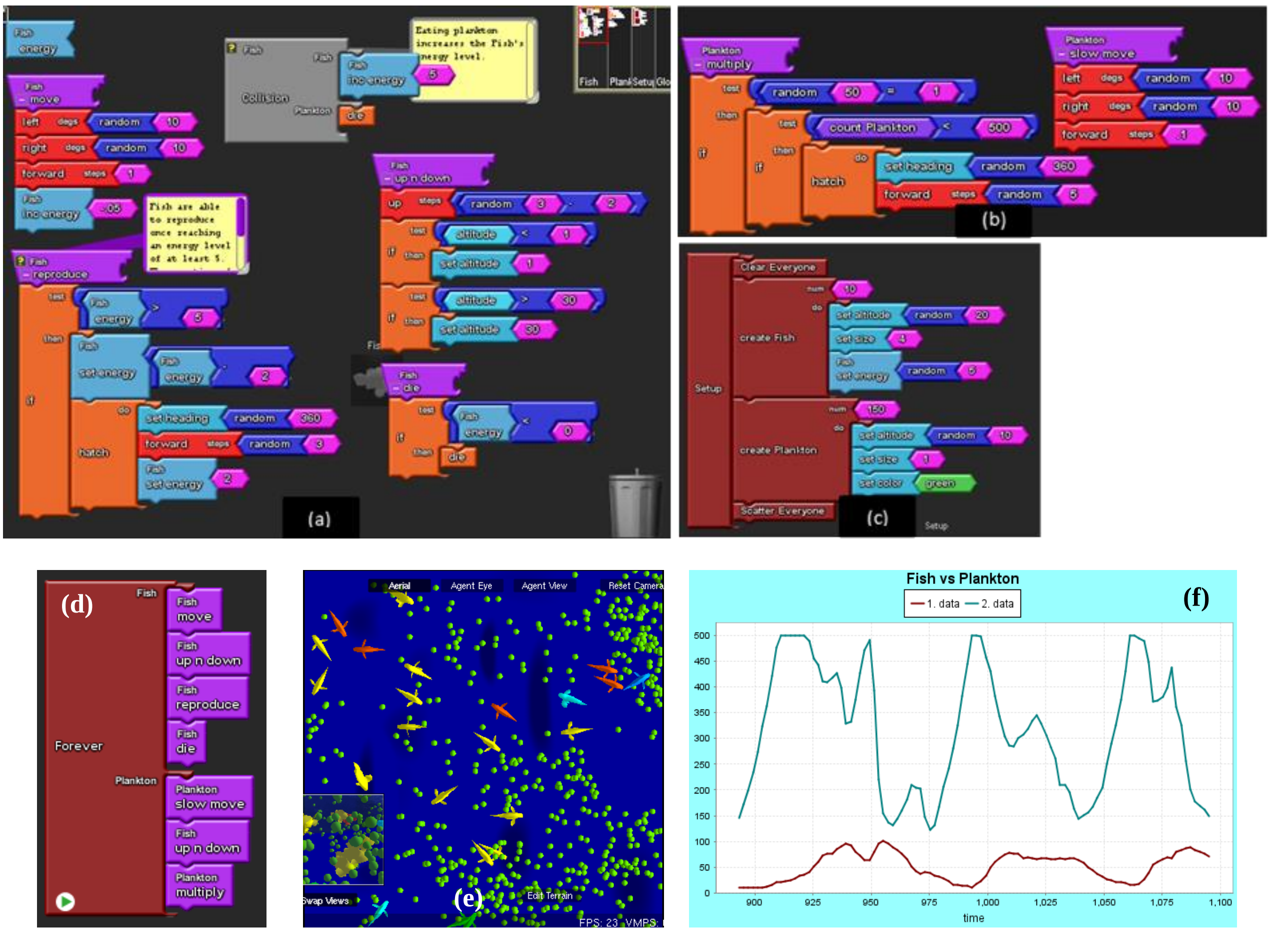


Figure 1. ABM For Fish Population. (a) Mathematical rules for fishes; (b) Math. rules for planktons; (c) Setting up Initial condition; (d) Coding for Execution; (e) Simulation Display, and (f) Graphical Results

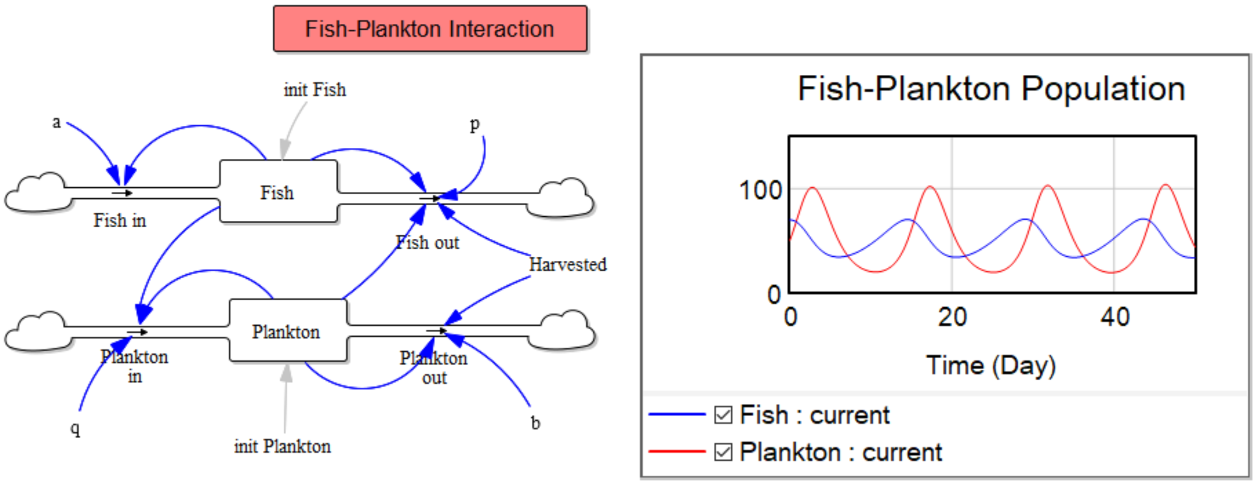


Figure 2. Fish-Plankton Interaction using SDM

In the Figure 1, which illustrates Agent-Based Modeling (ABM), the rules are assigned to all individual agents, including fish and plankton. In contrast, the Figure 2, representing Systems Dynamics Modeling (SDM), shows that the rules are applied to the fish and plankton populations, which are aggregate variables. Nevertheless, the general patterns observed in these two modeling approaches are quite similar, as both demonstrate oscillations between the fish and plankton populations.

3- Indonesia School Zone Regulation

Education in Indonesia is undergoing dynamic changes. These changes aim to improve the educational quality for our younger generations. However, many of these reforms have not been thoroughly analyzed, resulting in implementations that are not always effective and sometimes contradictory, even creating additional challenges. For instance, numerous adjustments have been made to the curriculum, yet these changes have not led to significant improvements in student achievement. Our students still rank in the lower tiers in assessments such as PISA and TIMSS. Efforts to enhance teacher quality have also been implemented, but since these improvements have been made only partially, no substantial progress has been realized [32].

The latest initiative aimed at enhancing educational service quality is the implementation of a zoning system for student recruitment, known as PPDB. However, this new recruitment system has sparked controversy in our society, with many students not being accepted into any school[*]. Additionally, numerous teachers have expressed concerns that the academic quality of their students has declined due to a decrease in the quality of incoming students. In the most recent recruitment cycle, one parent whose son was not accepted into the nearest school measured the distance from his home to the school, which was only a few hundred meters away. Nonetheless, his son was still not admitted to that school[†].

The school zoning system is based on Permendikbud No. 17 Tahun 2017. According to this regulation, state schools are required to accept students who reside within the closest zone radius of the schools. At least 90% of the total number of accepted students must come from this zoning system. The zone radius for students is determined by the minimum distance from their homes to a particular school. However, this criterion has been interpreted in various ways. Some districts use the distance from the students' homes to the school, while others use the distance from the students' villages to the school. Consequently, the criteria used by schools to accept new students have varied significantly. Furthermore, the method for determining the radius is done manually, making it prone to errors. In this article, the students' home addresses will be used to establish the minimal zone radius for their chosen school, as this criterion is the most intuitive for assessing a student's proximity to the school.

4- Mathematical Concepts Used to Determine Schools Zone

Determining school zones using various mathematical concepts is not conceptually difficult. The main challenge arises from the computations involved. This is because the computation needed for partitioning an area is an iterative process, which becomes computationally intensive as the number of schools (sites) increases [33, 34]. Essentially, determining school zone boundaries involves partitioning an area into several sections, with a school located in each section.

[*] https://regional.kompas.com/read/2018/07/11/17362241/sistem-zonasi-ppdb-dinilai-hambat-pendidikan-anak
[†] https://www.youtube.com/watch?v=wT5Klz-B3Ik

We will investigate four mathematical concepts for this partitioning: (i) perpendicular bisector, (ii) radical axis, (iii) Voronoi, and (iv) Agent-Based Modeling (ABM). With the exception of the Voronoi approach, the first three concepts will be explained in the following sections. As mentioned earlier, the Voronoi algorithm requires advanced mathematics to be understood. Therefore, in this article, we will apply the Voronoi approach using GeoGebra without further discussion. The implementation of ABM will be conducted using StarLogo TNG and Turbowarp [35, 36]. Turbowarp was chosen because its interface is similar to Scratch, making it familiar to secondary students, while also being significantly faster than Scratch.

4-1- The Use of Perpendicular Bisector to Partition an Area

The easiest method for partitioning an area is by using the perpendicular bisector in relation to certain points within that area. Suppose there are two points in the area; there will be one perpendicular bisector corresponding to those points, which will partition the area into two parts. All points located on the bisector will be equidistant from the two points. If there are three points in the area, there will be three perpendicular bisectors associated with those points, which will further partition the area into three parts, and so on. In this case, a point of intersection will emerge that is equidistant from the three points.

Definition: Suppose P is a set of n distinct points, called sites, in the plane. $V(P)$ is a subdivision of the plane into n cells such that:

1. Each cell contains exactly one site,
2. If a point q lies in a cell containing p_i then $d(q,p_i) < d(q,p_j)$ for all $p_i \in P$, $j \neq i$, where $d(x,y)$ is an *Euclidean distance* between point x and point y.

Examples:

1. If there is only one point on a plane, then the whole plane is the $V(P)$,
2. If P contains two points, then the $V(P)$ consists of two parts, determined by the perpendicular bisector of those two points,
3. If P contains three points, then the $V(P)$ consists of three parts determined by the bisector between each pair of these three points.

Figure 3 is the visualization of these three examples.

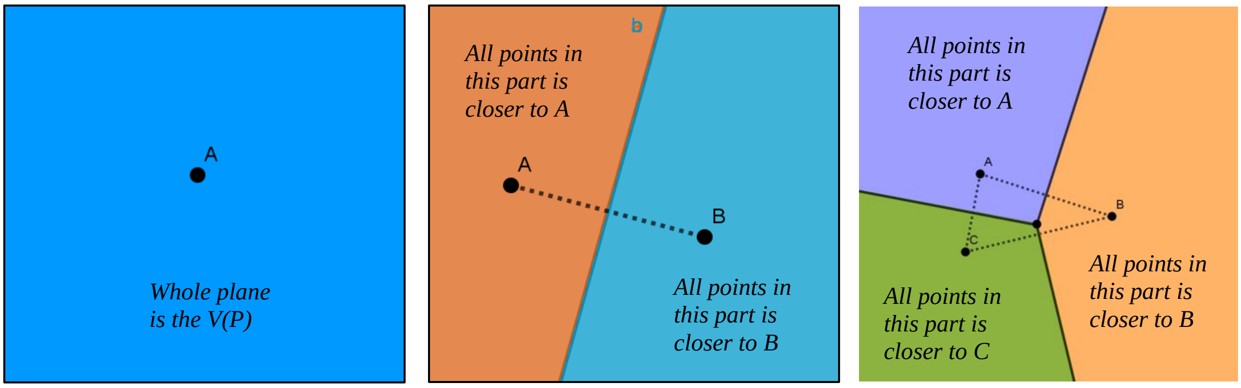


Figure 3. The partition of an area using one, two, and three points (sites)

If there are more than three points (sites) on the plane, the partitioning becomes more complicated, except in some special cases explained below. In Figure 4, special cases are illustrated: in the first case, all points are aligned in a straight line, and in the second case, all points form a circle.

In general, however, partitioning an area with more than three randomly located points is challenging to execute and visualize, even though the basic idea mentioned above can still be applied. This is where the concept of Voronoi comes into play. To create the partition, known as the Voronoi diagram V(P), for more than three randomly located points, the following steps are taken:

1. Draw triangles for each set of three points,
2. Find the center of a circle passing through every triangle,
3. Draw a bisector originated from the centre of the circle drawn in 2.

The following is an example of a partition generated by points A, B, C, D, E, F, and G.

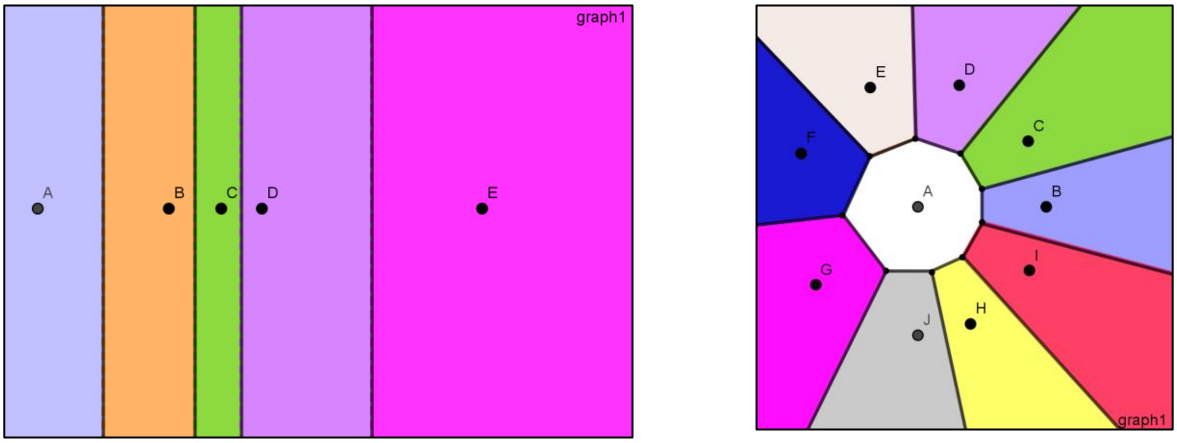


Figure 4. **Some partition that still can be drawn easily**

4-2- GeoGebra implementation of the Regular Partition

One method for implementing the procedure described above using GeoGebra is by defining new GeoGebra tools that will draw the circumcenter (the center of the outer circle) of any three points and construct the perpendicular bisector from that center (Figure 5). For the line connecting two circumcenters, the standard tool or command 'segment' in GeoGebra can be used.

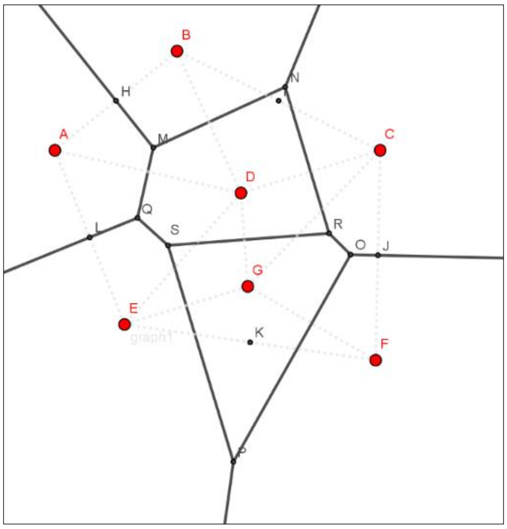


Figure 5. **An area partitioned by seven points A, B, C, D, E, F, and G**

Figure 6, shows some new tools which have been developed in GeoGebra to carry out those three steps explained above. The contents of the tools are GeoGebra scripts used to find (i) the power of three points, (ii) a power points, and (iii) power lines, which after being connected forms a triangle. The application of these tools to any set of three points given on Figure 7 results in partitioning the area where those points A, B, C, D, E, F, G, H, and I located.

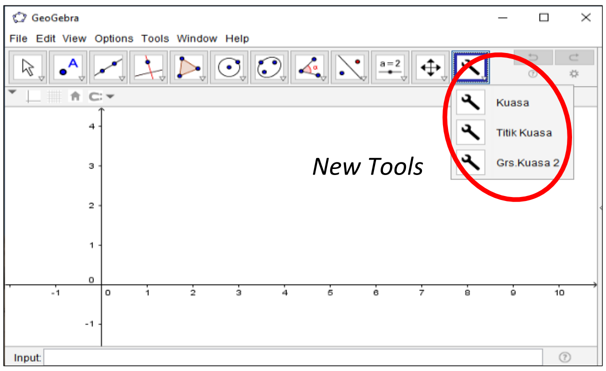


Figure 6. **GeoGebra new tools**

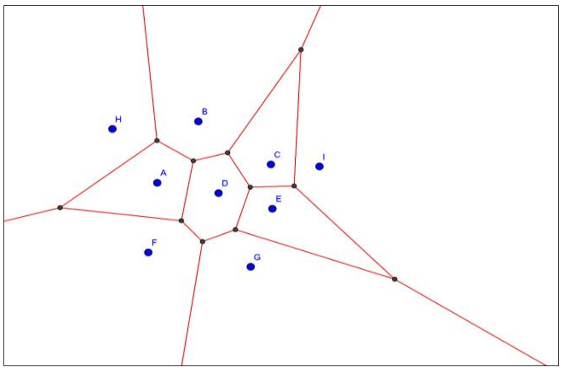


Figure 7. **Regular partition generated by 9 Points; A, B, C, D, E, F, G, H, and I.**

GeoGebra actually has a special command to draw the Voronoi diagram for a given set of points. Therefore, the result shown in Figure 7 can also be obtained using the GeoGebra command Voronoi {A, B, C, D, E, F, G, H, I}. It is important to note that in the obtained Voronoi diagram, all the points listed above have the same weight, which means the resulting Voronoi is referred to as a regular Voronoi or unweighted Voronoi.

5- Generalization of the Regular Partition

The procedure outlined above can be generalized by considering that points (or sites) may have different weights. This situation arises when a site, such as a school, has a maximum capacity for accepting new students. Consequently, each school may have a different weight based on its maximum capacity for enrolling new students. So, how should the partition be carried out to accommodate this requirement? One method to address this issue is by using the concept of the *Radical Axis* instead of a simple *Perpendicular Bisector*.

Definition: The *radical axis* of two circles is the locus of a moving point such that the lengths of the tangent lines drawn from the point to the two circles are equal (Figure 8).

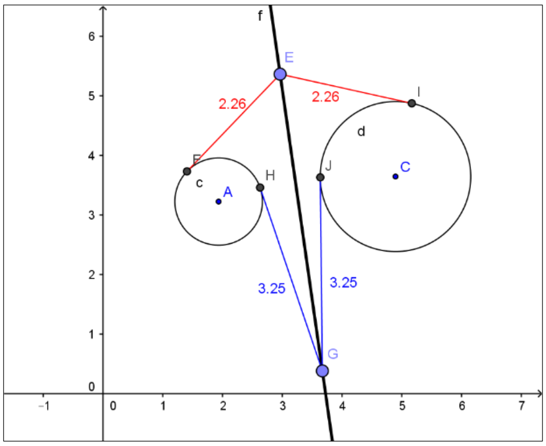


Figure 8. Radical Axis (black line)

For any two circles having equations:
$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0,$$
$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0,$$

it can be shown that their Radical Axis is: $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0.$

Now, if there are three circles on the plane, then there will be a point of intersection among three radical axis, called a *Radical Centre*.

In the diagram above, point G is the Radical Center of the three circles, meaning that point G has equal tangent lengths to all three circles. It can also be seen from the diagram that a Radical Center, along with its Radical Axes, can be used as a means to partition the plane into several regions.

Additional new GeoGebra tools have been developed to assist in generating either a Radical Center or Radical Axes. In Figure 9, the plane has been divided into three partitions, where three points with different weights are represented by the radii of their respective circles. Every point in the red region will be closer to circle *c*, while points in the blue and green regions will be closer to circles *d* and *f*, respectively. Figure 10 provides a further example of a plane partition generated using this concept, where six weighted points are located.

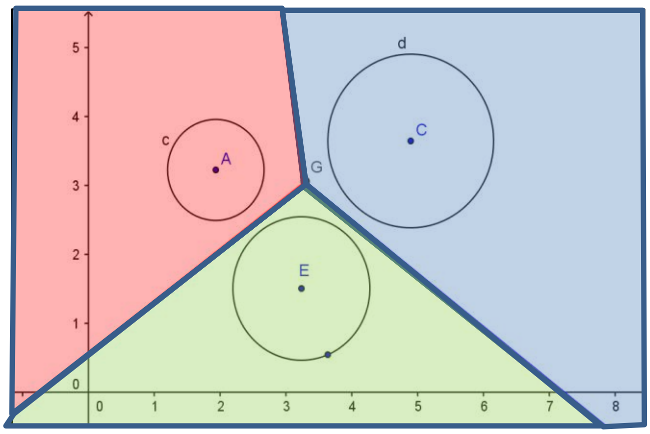


Figure 9. The Plane Partition using Radical Center

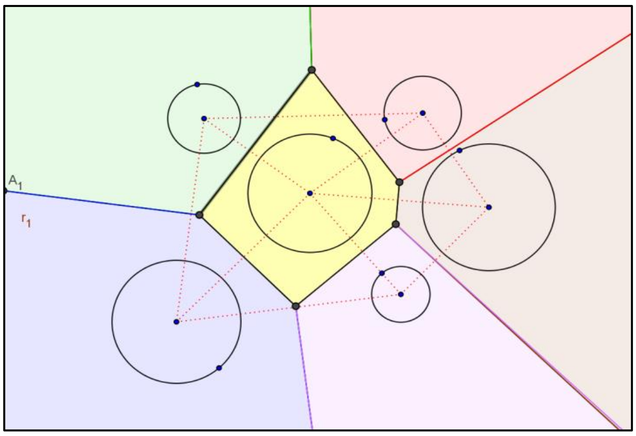


Figure 10. The Plane Partition into several regions

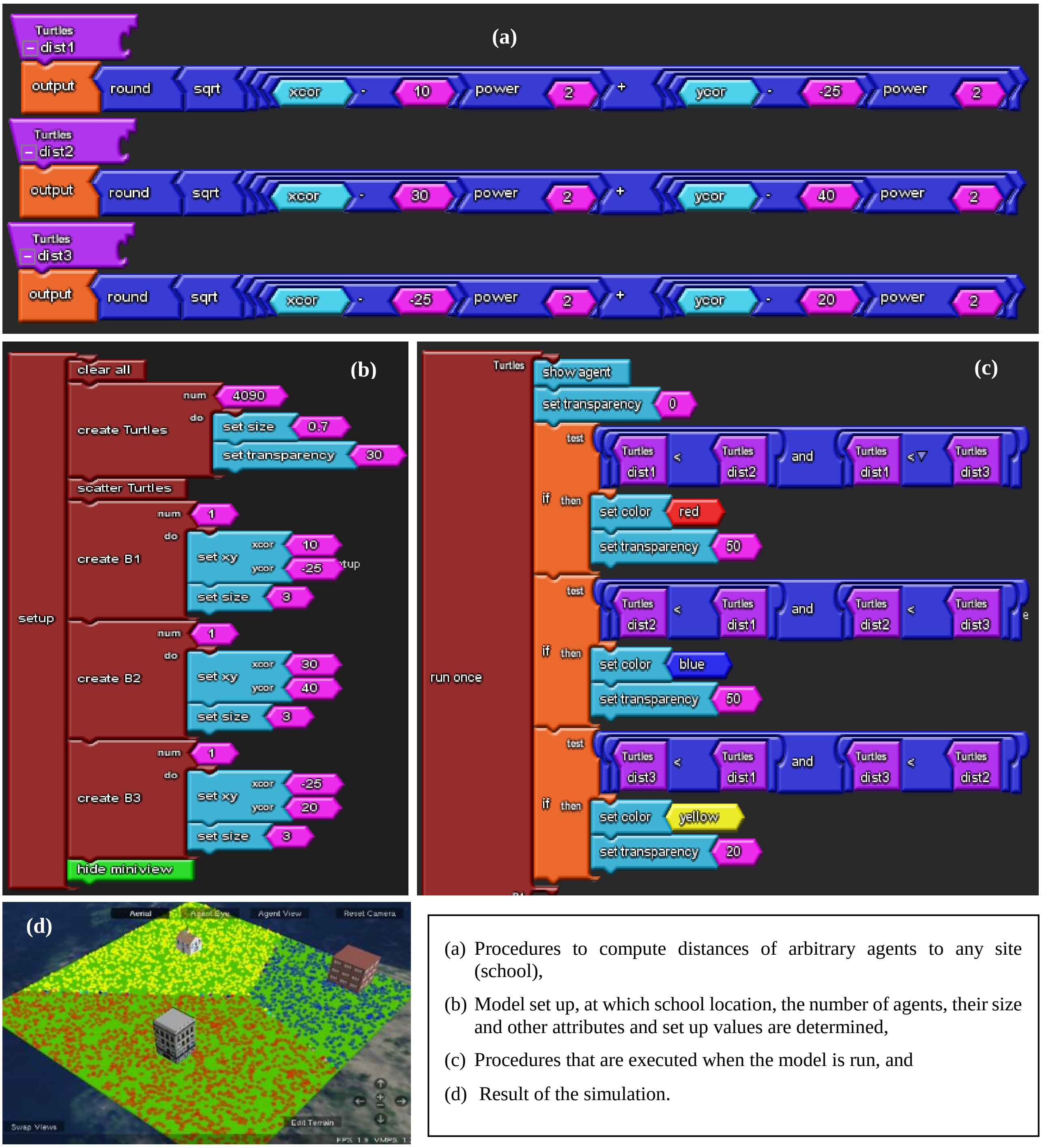
6- The Use of ABM

Partitioning an area using Agent-Based Modeling (ABM) is relatively simpler compared to the methods described above. This is because the partitioning process in ABM follows a very intuitive approach. For example, if there are three schools, their zones can be determined through the following steps:

1. Create agents and classify them into four breeds, that is three schools and students,
2. Create, say, 5000 students, and three schools,
3. For every agent, give rules to be executed when the program is run, namely (i) determine its distance to every school, and (ii) color it according to the color of the associated nearest school.

Clearly, here students only need to use the concept of inequality and Euclidean distance, that is, the Pythagorean formula.

In StarLogo TNG, the above steps are implemented as follows (Figure 11):



(a) Procedures to compute distances of arbitrary agents to any site (school),

(b) Model set up, at which school location, the number of agents, their size and other attributes and set up values are determined,

(c) Procedures that are executed when the model is run, and

(d) Result of the simulation.

Figure 11. ABM implementation of an area partitioning

It can be seen from Figure 11 (d) that this ABM model is successfully able to partition the area into three parts with a clear border where every part is represented by a different color.

7- Results of determining School Zones border of 6 SMPN in Singaraja

The tools that have been used so far are now applied to determine the school borders of seven Junior State Secondary Schools (SMPN) in Singaraja, Bali, Indonesia. GeoGebra and Agent-Based Modeling (ABM) will be utilized to establish these borders. The procedures are as follows:

- Place a Google Map of the city of Singaraja, along with the locations of the SMPN schools, into GeoGebra.
- Identify the positions of the schools under consideration and mark a point for each school.
- Divide the area using the established tools into several parts, ensuring that each region contains one school.

7-1- Case of Regular Partitioning

In regular partitioning, the method of perpendicular bisector is used. The following diagram is a result of applying this method (Figure 12).

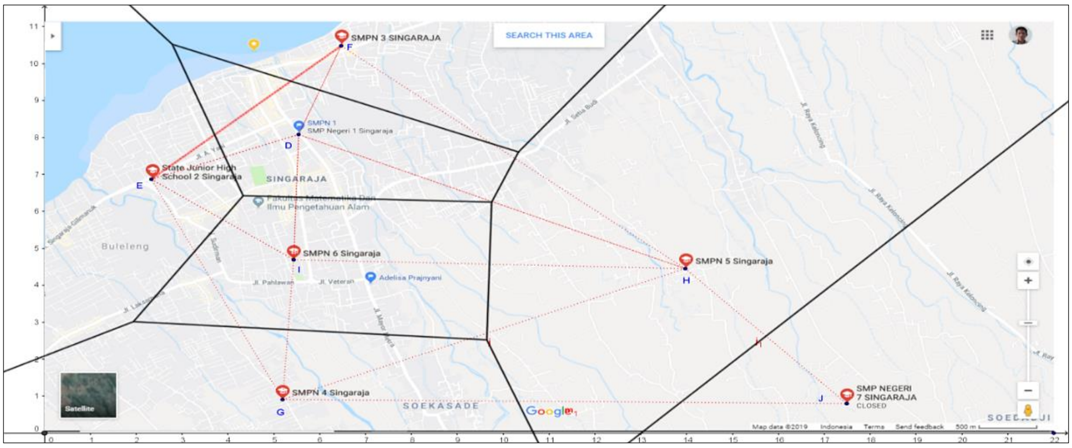


Figure 12. Zone of several SMP Schools in Singaraja

Based on the obtained result, further decisions can be drawn regarding the border of each school. It should be reminded that points on the border will have a similar distance from their associated schools.

7-2- Case of Weighted Partitioning

If each school has a different weight, the partition diagram will appear quite different. In this context, the schools' capacities to accept new students have been used to determine their weights. For example, SMPN I Singaraja, which has a capacity to recruit 267 new students (10.6% of the total new students), is assigned a school radius of 1.1. In contrast, SMPN IV, with a capacity of 364 new students (14.5% of the total newcomers), is assigned a radius of 1.5, which is the same as that of SMPN VI. These weights are calculated by multiplying the schools' recruitment capacity fractions by 10. The choice of 10 is made purely to enhance the clarity of the partition graph. In summary, the following table presents the school capacities for recruiting new students and, consequently, the schools' weights (Table 1).

Table 1. Capacity of State Junior High School to accept new students in the academic year 2019

No.	School	Capacity	Fraction	Weight (Radius)
1	SMP Negeri I Singaraja	267	0.106	1.1
2	SMP Negeri II Singaraja	560	0.222	2.2
3	SMP Negeri III Singaraja	484	0.192	1.9
4	SMP Negeri IV Singaraja	364	0.145	1.5
5	SMP Negeri V Singaraja	353	0.140	1.4
6	SMP Negeri VI Singaraja	364	0.145	1.5

Although the determination of this weighted partition again can be done by using GeoGebra through the development of some new tools, for the purpose of motivating and encouraging students to study modelling and coding, the ABM approach will be used. The implementation of this ABM approach is very intuitive, they are:

1. Determine the distance from each agent to the schools and divide each distance by the weight of the associated school.
2. Change the direction of each agent toward its nearest school.
3. To find the border, move in the opposite direction of what was determined in step 2.
4. Repeating step 3 will produce the emergent pattern depicted in Figure 13.

From the diagram, it is clear that this pattern differs significantly from the one observed in the case of a regular partition. Schools with larger weights will have larger zones, while those with smaller weights will have smaller regions. In contrast to the earlier ABM method, where the code aimed to find the area of the regions and the border was formed by the common edges of adjacent areas, this code seeks to identify the borders of the regions.

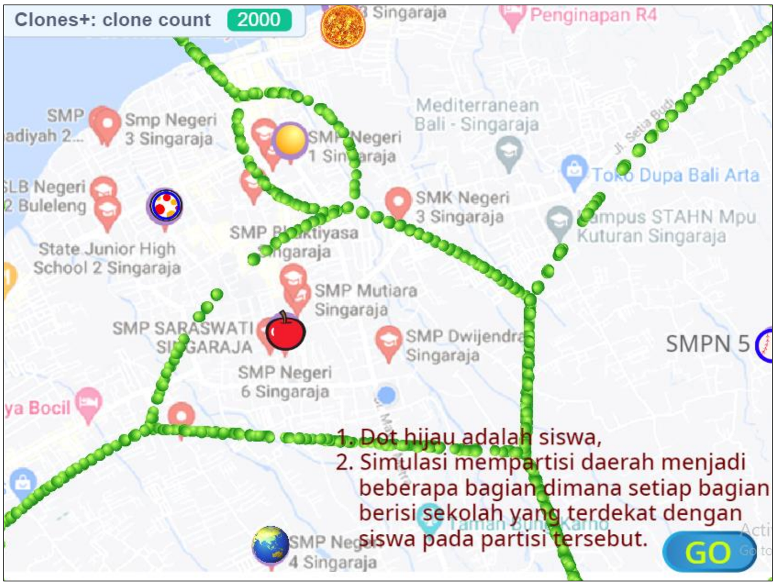


Figure 13. Zone of several SMP Schools in Singaraja with weighted capacity

8- Conclusion

This article has demonstrated how GeoGebra and Agent-Based Modeling (ABM) can be used to partition an area into several parts. Area partitioning has many applications, one of which often causes tension during the recruitment of new students in Indonesia: determining school boundaries. These school boundaries play a crucial role in deciding the eligibility of new students. If a student is located within a specific school zone or boundary, they meet the criteria for enrollment in that school. Therefore, using this pressing issue as a problem for a modeling course is not only interesting but also motivating and challenging for students.

Utilizing school zone boundaries as a framework for a modeling course encourages rich exploration, as finding solutions involves numerous mathematical concepts from various branches, including algebra, geometry, analytic geometry, and even computational geometry like Voronoi diagrams. In this article, a new method has been proposed to address this partitioning problem using ABM. This approach seems intuitive for secondary school students, as they can position themselves as agents and apply rules based on their intuitive understanding. For example, when determining the area of a partition, students might (i) assess their distances as agents to all sites/schools and (ii) color themselves according to the site/school that is closest to them. By using this method, the area will be partitioned into several subareas represented by different colors corresponding to the sites/schools.

Moreover, employing the ABM approach will enhance students' Computational Thinking (CT) skills, as many CT components, such as decomposition, pattern recognition, abstraction, and algorithm development, are addressed through this methodology. Therefore, the problem of determining school zone boundaries is an excellent topic for conducting a modeling course.

9- Declarations

9-1- Data Availability Statement

The data presented in this study are available on request from the corresponding author.

9-2- Funding

The author thanks The Ganesha University of Education for their financial support to publish this article.

9-3- Institutional Review Board Statement

Not applicable.

9-4- Informed Consent Statement

Not applicable.

9-5- Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the author.

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