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Fear and Group Defense Effect of a Holling Type IV Predator-Prey System Intraspecific Competition

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Abstract

Field and experimental data on aquatic ecosystem species show the effect of fear on changing prey demographics. The fear effect has an impact on aquatic ecosystems, such as species migration to settled areas. In this paper, the type of research described is a literature study. The cost effect assigned to the reproductive system of the prey population and the predation function response are given as Holling Type IV for research purposes to model the fear effect. Some research novelties, the equilibrium points are all shown in the population dynamics system model with an analysis of positive equilibrium. Positive and biologically realistic equilibrium points were analyzed using the Routh-Hurwitz criterion which is mathematically a local asymptotically stable. A pair of imaginary eigenvalues with a negative real part can increase population growth. An equilibrium region showing equilibrium for several parameters such as extinction, no predators, and two populations coexisting in a sustainable manner. The correlation and fluctuation of fear and fear cost were investigated to obtain a better model. The results of the numerical simulations show that the prey population becomes more daring to fight or fighting power with significant prey growth rates or high predator mortality rates.

Keywords:

Equilibrium Points;

Functional Response;

Holling Type IV;

Predator-Prey System;

Simulation; Stability.

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1- Introduction

Predator-prey interactions are a major topic in biological, mathematical, and ecological research. Mathematical modeling provides preliminary knowledge of complex ecological events [1–3]. Predatory interaction mechanisms in the response process are easy to find in natural ecosystems. Predator hunting methods, both direct and indirect, resulted in the extinction of species from population groups [4]. Predators play a large role as the most powerful factor in the predator-prey response model [5]. Changes in behavior, habitat migration, breeding control, and species population extinction are the effects of predatory forces in predator-prey interactions [6, 7]. In prey populations, the response is different in the face of predatory species. Each prey species provides a different anti-predator response from each predation given by the predator [8]. Several types of anti-predator responses can be observed in ecosystems, such as migration of native habitats to protected habitats, decreased foraging activities resulting in reduced detection by predators, and living in defensive groups that can withstand pressure from predatory interactions [9]. Resistance to prey

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interaction in predators is highly dependent on the number of prey populations that unite in groups against predatory interactions [10, 11]. The formation of anti-predator survival groups is also realistically calculated in the population mathematical model. Previous studies revealed that strong anti-predatory properties outnumbered large groups of predators. The nature of survival in groups certainly provides an advantage but is contrary to the early detection of prey [12]. Predators will easily detect large groups of prey. Meanwhile, predators continue to develop their ways of preying on surviving prey through psychological development. Such conditions often occur as a form of balance for the survival of the species [13, 14].

Prey with anti-predatory behavior that forms groups in large numbers can increase survival rates because of the mutual protection between group members [15]. Attacks and defenses against predators and prey are common in fish, birds, vertebrate, and invertebrate species. The Cape Buffalo takes anti-predatory action against hyenas, generally starting when the prey equals the predator population. Vespa affinis is a species of deadly wasp that preys on the Argyrodes spider. In general, changes in prey behavior in predator-prey interactions are only carried out in fast-growing prey species [16–18], which occur because of the nature of their fear of their prey. The greater the pressure from predatory predation, the greater the effect of fear on the prey [19]. The behavior occurs when predator and prey face each other during the predation process. Prey can change their behavior immediately after encountering a predator. The predatory style of the predator can change significantly when the prey feels threatened and frightened [20, 21]. In analyzing this, many mathematical models only take direct predation into account, although many things can happen during predator-prey predation. The effects of fear are realistic and important to consider in population growth studies [22]. Predatory attacks on their prey can have psychological and behavioral effects. These changes had a significant impact on the overall prey demographics. The effects of fear can also cause long-term damage to prey species [23], such as reductions in population growth and diet due to increased awareness of prey species. Starvation and death in prey species can end the story of the effects of persistent fear [24, 25].

Predator-prey interactions are a major factor in mathematical modeling [26]. Many formulations of predation functions have been conceptualized, from simple to complex forms. Predatory characteristics determine the formation of the predation function in the dynamic model. The Holling function response is representative and realistic for the predation function [27]. The first predation response function is Holling Type I, characterized by a monotonically increasing predation function. Species with this response function had the same population ratio and predation rate [28]. The larger the prey population, the more prey the predator will eat [29]. Ecologically, predatory species with Holling Type I responses tend to wait for prey. The following predation functions are Holing types II and Holing Type III, which have almost the same characteristics. However, the ratio of population numbers does not affect this characteristic because predators will be equally saturated at a certain time. The most visible difference is the initial process of predation. Holling Type II tends to accelerate more quickly, while Holling Type III tends to take longer. Ecologically, the characteristics of Holling Type III require time to supervise their prey first. Thus, the considerations for determining the response function are highly dependent on the ecological factors of the species [30]. A study in Alsakaji [31] introduced Holling Type IV for the effect of fear. The most common and straightforward fear effect behavior is the Holling Type IV or Monod-Haldane predation function [32]. A characteristic feature of Holling Type IV species is that they cannot survive at the upper limit of the prey density threshold [33, 34]. Figure 1 compares the response function trajectories in Holling types I–IV with a simple form of the predation process.

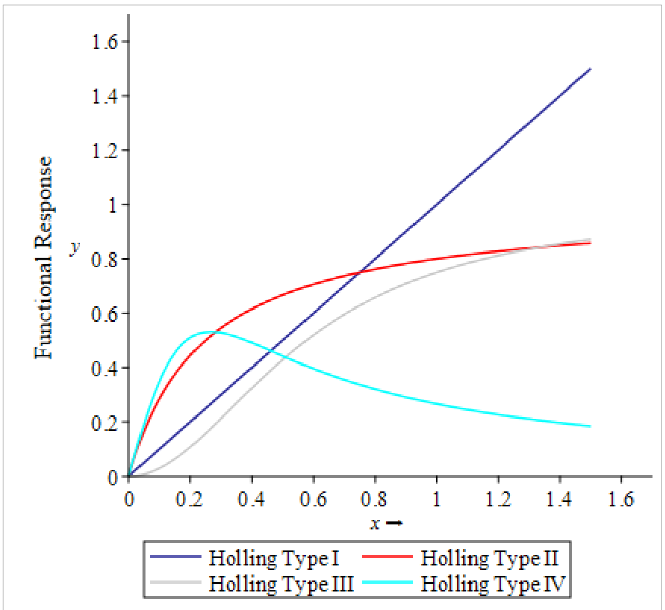

Figure 1. Trajectories Holling response function

Another factor considered in the preparation of mathematical models of population dynamics is the behavior of species that interact rationally intraspecifically. Intraspecific interactions with predator-prey can occur in a symbiotic mutualism. Helping behavior and mutualism symbiosis appear naturally based on species, such as interactions in bee species, where the bee colony consists of worker bees, army bees, and queen bees. Worker bees are in charge of collecting honey for the army bees and queen bees; soldier bees are in order of protecting the hive; and the queen bee is in charge of leading the colony to work. However, this intraspecific interaction can cause harm to the colony population. One of the essential characteristics that can encourage competitive behavior among predator groups is a sense of competition, which plays a vital role in predatory interactions when capturing prey. The large ratio between predator and prey species, which continues to experience the effects of fear with a declining population, makes food battles unavoidable. The rarity of prey species is realistic to mention in the assumptions of the population dynamics model. The main thing in this study is to formulate the effect of fear on population growth and the impact of mortality rates that allow intraspecific predator competition to occur. In this paper, investigations are carried out for several population model variables, such as the effects of fear fluctuations and intraspecific fluctuations. These two variables are realistic to consider in the model and significantly impact the population model.

2- Basic Predator-Prey Model

This study, using a research flow or a simple research methodology. This is done to support the research steps that are easy for the author to take. Research methodology refers to the research flow chart as follows (Figure 2).

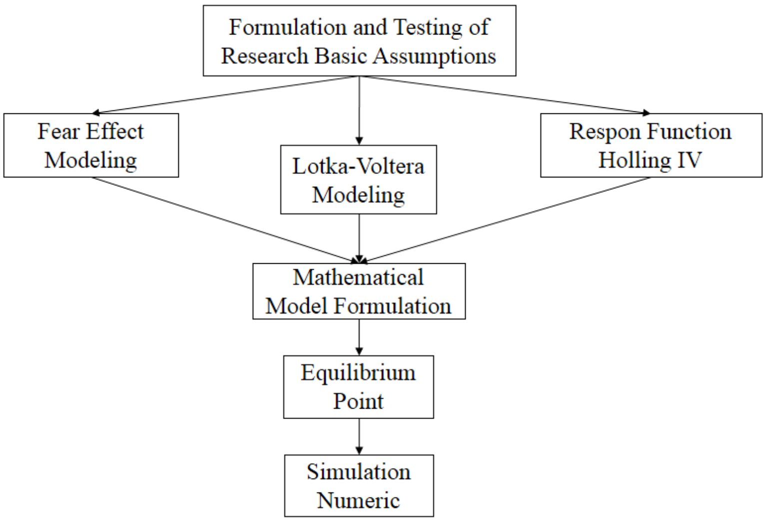

Figure 2. Research flow chart

In the section on formulation assumptions, the author tries to make assumptions that are very similar to the actual conditions in the life of living ecosystems. The focus of the assumptions built is the fear effect, Lotka-Voltera, and Holling Type IV. The assumptions built also come from relevant previous research. Furthermore, at the formulation stage of the model, the concept of mathematical differential equations is given to find solutions for the model. Meanwhile, at the equilibrium point and numerical simulation stages, it is the final stage to obtain research results and test the truth of these results. Tests are carried out mathematically and realistically for real conditions.

The composition of the model was prepared by considering the predator-prey model, fear effects, and intraspecific interactions. The growth of predator and prey population density follows the concept of logistic growth without predator population. The de-scription of logistic growth in prey species consists of prey growth rate, natural prey mortality rate, and mortality rate from population density due to intraspecific competition. Variable x and y represent the population density of predators and prey, respectively. The growth of each population is defined in time t, so that each variable can be expressed as $x(t)$ and $y(t)$. In simple terms, the mathematical model of prey population growth can be described as follows

$$\frac{dx}{dt} = rx - \delta_1 x - hx^2, \tag{1}$$

where r is the birth rate of the prey population, δ_1 is the death rate for prey population, and h is the rate of intraspecific prey interaction.

Experimental research shows that prey behavior after experiencing the effects of fear decreases the growth of prey species. This behavior is widely supported by relevant ecological and scientific research. From the basic concept of the study, the predator-prey model on population growth involves a growth variable with a fear effect function f(α,β,y), so Equation 1 can be written as;

$$\frac{dx}{dt} = rxf(\alpha, \beta, y) - \delta_1 x - hx^2, \tag{2}$$

where β and α represent the minimum fear cost and fear level, respectively. The fear effect function shows the composition of parameters that define the level of fear. This parameter is highly dependent on the fear behavior of the prey species. The predation function as a predator-prey model composition is included in p(x) in Equation 2, so the population with the fear effect model can be expressed as

$$\begin{aligned} \frac{dx}{dt} &= rxf(\alpha, \beta, y) - \delta_1 x - hx^2 - p(x)y, \\ \frac{dy}{dt} &= p(x)y - my^2 - \delta_2 y, \end{aligned} \tag{3}$$

where δ_2 and m represent the natural mortality rate of the predator populations and conversion rates of predatory intraspecific interactions, respectively, and p(x) is the functional response of the predator species to the prey population density per unit time.

Each predatory species has predation characteristics in the ecological environment. The species that is widely considered in terms of predation characteristics is Holling. Holling Type IV is used as a consideration for selecting the functional response; the mathematical model can be expressed as follows

$$p(x) = \frac{\rho x}{\sigma + \tau \sigma x + x^2}, \tag{4}$$

where ρ, τ and σ are non-negative constants with realistic considerations according to actual conditions in the ecosystem.

Mathematically the prey model will be developed in the form of a function with a fear effect based on the assumptions that have been given. A function with a fear effect will be assigned to the growth of prey-one and prey-two species as f(α,β,y). The σ and β parameters represent the level of prey fear (anti-predator behavior) and the minimum prey fear strength, respectively. Fear effect functions f(α,β,y) can be described mathematically as $f(\alpha,\beta,y) = \beta + \left(\frac{\alpha(1-\beta)}{\alpha+y}\right)$ where β has the condition $\beta \in [0,1)$. In the fear effect function, it is assumed that f(α,β,y)=β, f(α,β,y)=1, $\lim_{y\to\infty} f(\alpha,\beta,y) = \beta$, and $\lim_{y\to\infty} f(\alpha,\beta,y) = 1$. The function f(α,β,y)=β shows that the prey population is always less than the minimum fear power β. The f(α,β,0)=1 function indicates that if there is no predator population, the fear function does not affect the growth of the prey population. The fear function $\lim_{y\to\infty} f(\alpha,\beta,y) = \beta$ indicates that as the predator population increases, the prey population experiences minimum fear stress from the predator species. The fear function $\lim_{y\to\infty} f(\alpha,\beta,y) = 1$ shows that after the fear level is saturated at a certain point in the prey population, the fear function has no effect due to the physiological impact when the prey is accustomed to the predator threat. Such events often occur in predator-prey interactions. The composition of each parameter will be given, taking into account the model's dimensions. Table 1 summarizes the parameters in the model with each variable and its units/dimensions.

Table 1. Description of the parameters and variables in the model

Symbol	Description	Units
x	The density of prey population	$[N]$
y	The density of predator population	$[N]$
r	The birth rate of the prey population	$[T]^{-1}$
α	Level of fear	$[N]$
β	Cost of minimum fear	-
δ_1	The natural death rate of prey	$[T]^{-1}$
δ_2	The natural death rate of predator	$[T]^{-1}$
m	coefficient intraspecific prey	$[T]^{-1}[N]^{-1}$
h	coefficient intraspecific predator	$[T]^{-1}[N]^{-1}$
ρ	The rate of predation	$[N][T]^{-1}$
σ	The half-saturation constant	-
τ	Tolerance limit of predator	-

Therefore, our final model becomes,

$$\begin{aligned} \frac{dx}{dt} &= rx\left(\beta + \left(\frac{\alpha(1-\beta)}{\alpha+y}\right)\right) - \delta_1 x - hx^2 - \frac{\rho xy}{\sigma + \tau\sigma x + x^2}, \\ \frac{dy}{dt} &= \frac{\rho xy}{\sigma + \tau\sigma x + x^2} - my^2 - \delta_2 y, \end{aligned} \tag{5}$$

where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. The model system in Equation 5 is subjected to a positive initial value $x(0) = x_0 \geq 0$ and $y(0) = y_0 \geq 0$ from the population model.

3- Results and Discussion

At this stage, the research results are described in detail and structured according to the research stages. Two things that will be discussed are balance analysis and simulation of research programs.

3-1- Equilibrium Analysis

The equilibrium point is analysed using the basic concept of solving differential equations. The model in Equation 5 is the basis for exploring the development of the equilibrium point. Ten balances are obtained from Equation 5, namely eight real balances and two negative imaginary balance points. All these equilibrium points are taken from the solution of the differential equation model, where the steady-state solution model in Equation 5 is obtained from two solutions, i.e., trivial and non-trivial. Of all the equilibrium points obtained, only two points are realistic to be considered as follows: $E_0 = (0,0)$, $E_1 = (0, B_1)$, $E_2 = (A_2, 0)$, $E_3 = (A_3, B_3)$, $E_4 = (A_4, B_4)$, $E_3 = (A_5, B_5)$, $E_6 = (A_6, B_6)$, $E_7 = (A_7, B_7)$, $E_8 = (A_8, B_8)$, $E_9 = (A_9, B_9)$, $E_{10} = (A_{10}, B_{10})$. Mathematically rational in the all equilibrium model there are only four non-negative equilibrium. These four equilibria are realistic in accordance with the assumptions to be made. The four points are $E_0 = (0,0)$, $E_2 = (A_2, 0)$, $E_3 = (A_3, B_3)$, and $E_5 = (A_5, B_5)$.

Each non-trivial solution in Equation 5 is used to balance the septic polynomial equation model for the equilibrium value x* as follows;

$$a_7 x^7 + a_6 x^6 + a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \tag{6}$$

There are seven roots of the Equation that are components of the equilibrium point. Only two points that satisfy the local optimal point are positive roots. Only one positive root is selected to develop the equilibrium value in the numerical simulation. While the value of the equilibrium point y* is highly dependent on

$$y^* = \frac{b_2 x^2 + b_1 x + b_0}{c_2 x^2 + c_1 x + c_0} \tag{7}$$

Analytically complex to show in detail, application assisted solution program is used in this research. The root property of a septic polynomial can be characterized as the following discriminant

$$\Delta = \frac{1}{a_7} \det(R) \tag{8}$$

where the Sylvester matrix characteristic is given by;

$$R = \begin{bmatrix} a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & 0 & 0 \\ 0 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7a_7 & 6a_6 & 5a_5 & 4a_4 & 3a_3 & 2a_2 & a_1 \end{bmatrix} \tag{9}$$

If $\Delta>0$ then the polynomial has seven distinct roots or has three pairs of complex conjugate roots and one real root. If $\Delta=0$ then the polynomial has at least two complex roots. If $\Delta < 0$ then there are six real roots and two complex roots.

The realistic positive real root solution to be considered becomes the basis for testing the stability of the model in Equation 5. Stability testing uses criteria that support the re-search. The formed Jacobian matrix shows characteristic equations. The Jacobian matrix used is $J_{E_{10}} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$. After obtaining a Jacobian matrix that corresponds to Equation 5 with equilibrium E_{10}, the characteristic Equation of matrix $J_{E_{10}}$ can be obtained as follows;

$$\lambda^2 + b\lambda + a = 0 \tag{10}$$

The characteristic Equation shows realistic equilibrium stability. Local stability was obtained to achieve the Routh-Hurwitz criteria, i.e., $b > 0$, $a > 0$, and $b > a$. The equilibrium $E_5 = (A_5, B_5)$ is said to be asymptotically stable. The local stability obtained in this study suggests that the model in Equation 5 is viable in the long term. Many studies are against taking non-negative equilibrium only for equilibrium values. Many research statements require that the entire equilibrium point be taken into account in the form of the model that is compiled. The author takes a non-negative equilibrium by considering the sustainability of the life of the two populations, namely predators and prey. Meanwhile, if only one population is maintained, then in ecological science there is an imbalance of ecosystems in the environment. The imbalance in the ecosystem will have an impact on natural disasters or the death of species in general. Government or other policy authorities may consider models for conserving species. Intervention in the model is given by changing the value of parameter σ, to show a shift in the equilibrium point. An explanation of this is studied in more detail in numerical simulations.

3-2- Numerical Simulation

The numerical simulation requires taking realistic parameters to represent dimensions in Equation 5. The retrieval of parameter figures is based on relevant assumptions and references. Different parameters will be considered to affect the population dynamics system. Several types of behavioral models with each species are part of the analysis of the model used. The obtained parameters and other system parameters are assumptions. The following are the parameters taken from Equation 5: r=1.25, α=0.005, β=0.5, h=0.3, m=0.0008, δ_1=0.4, δ_2=0.2, ρ=0.5, τ=0.5, and σ=0.25. These parameters are simulated to obtain a realistic positive equilibrium to consider $E_{10} = (A_{10}, B_{10})$ where $A_{10} = 0.1104617645$ and $B_{10} = 0.1312357995$. Equilibrium gives the characteristic $\lambda^2 + 0.000330608554\lambda + 0.04860540815 = 0$ from the form of the characteristic in Equation 5. This shows that the Routh-Hurwitz criteria are met so that the corresponding eigenvalues λ_1=-0.00016530427 and λ_2=-0.00016530427 can be obtained. The corresponding eigenvalues show real negative results equilibrium point $E_{10} = (A_{10}, B_{10})$ satisfies local asymptotic stability. The following analysis is to see the movement of the trajectory of each species at Equation 5. The trajectory is presented in a numerical analysis simulation by considering two important parameters, i.e., the level of fear and the cost of fear. These two parameters significantly affect the sustainability of the model for ideal population growth. Parameter changes still pay attention to the stability of the model as an absolute requirement so that the model in Equation 5 can be maintained. The fear level and fear cost parameters in the monotone simulation decreased separately.

Figures 3 to 7 shows the change stability of the model movement in Equation 5 for each prey and predator species. The initial value taken from Equation 5 are x(0)=0.061 and y(0)=0.071. In situations where it is possible to change the fear, movement can still be categorized as stable. Simultaneous changes continue to move to-wards stability for a long time. The smaller the level of fear given, the softer the fluctuations towards stability. This finding is in line with the study results that the lower the level of fear, the growth of the prey population will continue to exist towards long-term stability. Theoretically, this is very supportive of the condition of the ecological system of predator-prey species. A low level of fear will certainly allow prey species to live in good natural conditions without pressure from predators. Prey species are freer to find food, breed, and build sustainable ecosystems. The subsequent analysis is the cost of the fear parameter, which moves the same as the decrease in the fear level parameter. The adapted fear cost is the fear cost that satisfies the stability of the model in Equation 5, as shown in Figures 3 to 12.

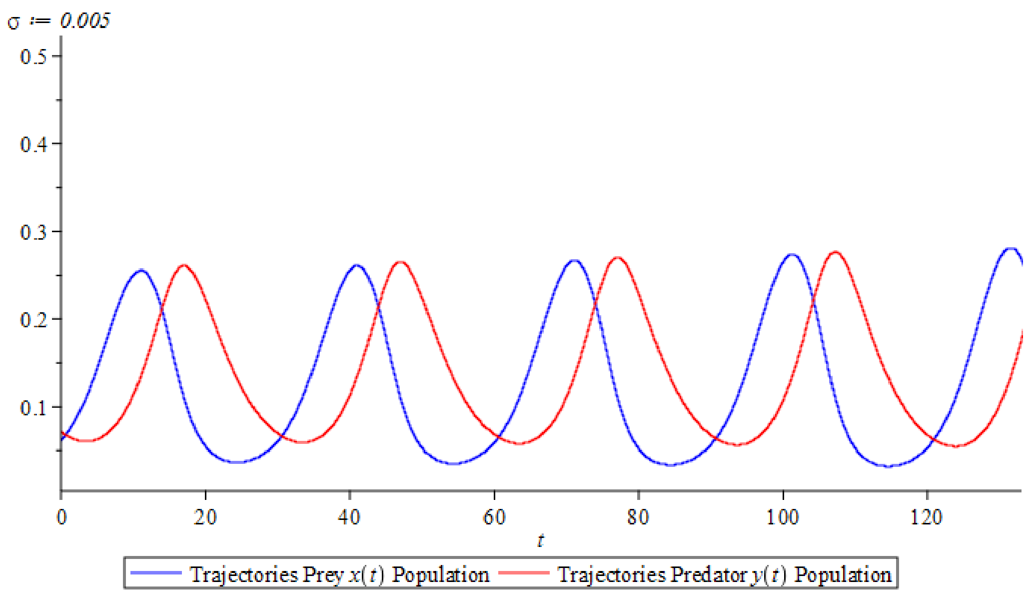

Figure 3. Trajectories fear level σ=0.005

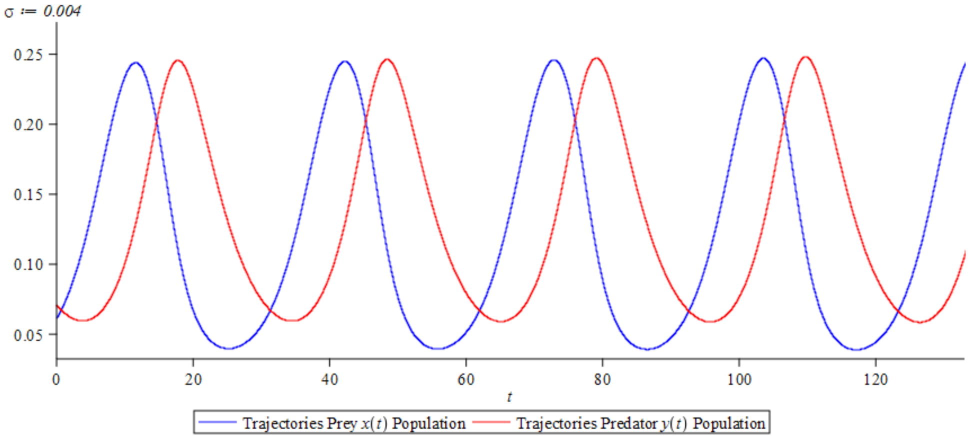

Figure 4. Trajectories fear level σ=0.004

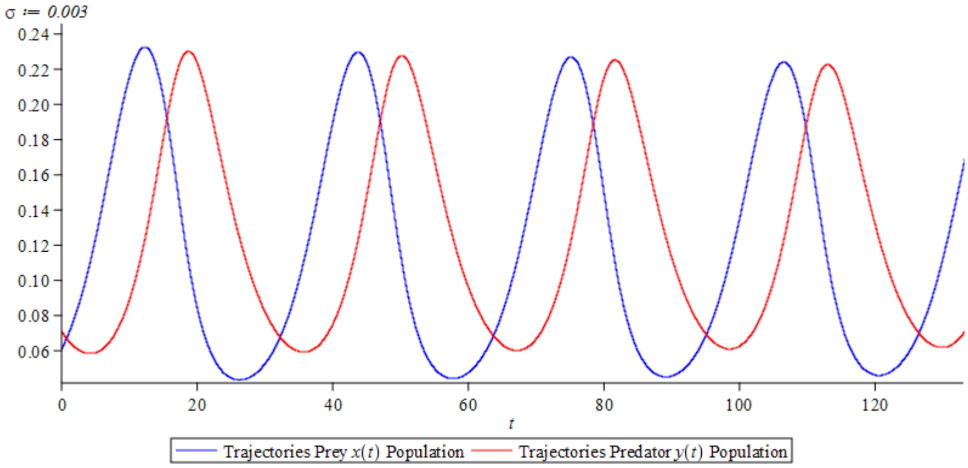

Figure 5. Trajectories fear level σ=0.003

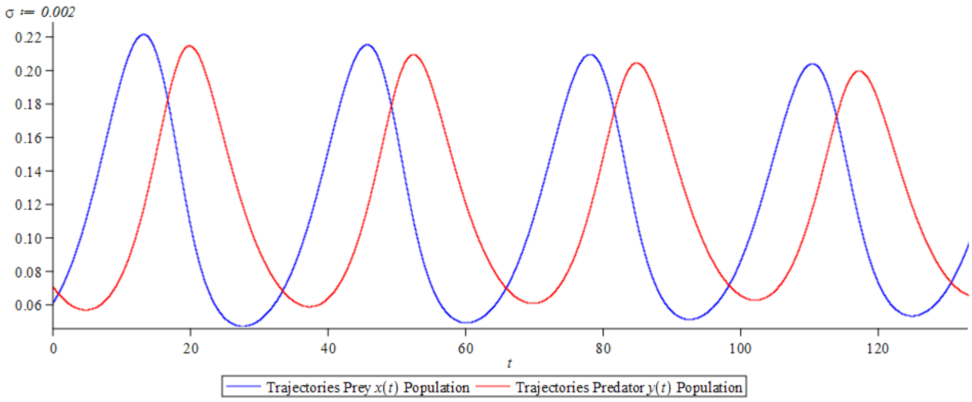

Figure 6. Trajectories fear level σ=0.002

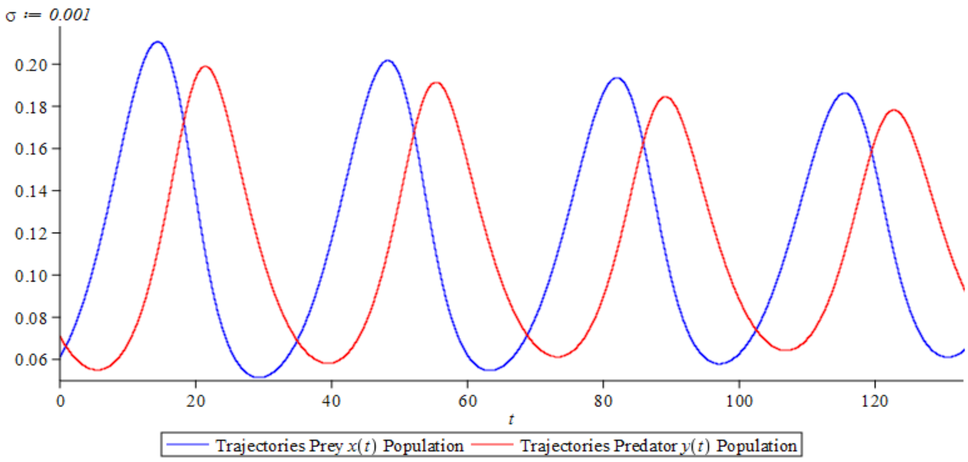

Figure 7. Trajectories fear level σ=0.001

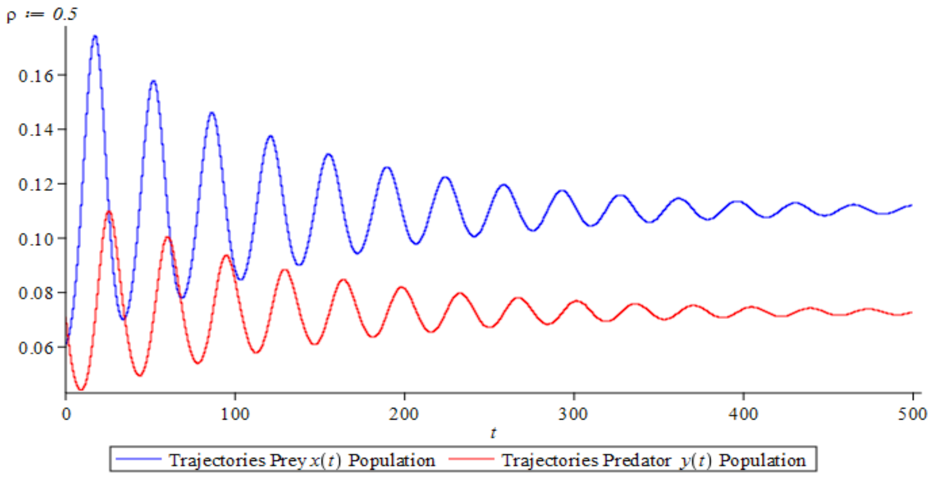

Figure 8. Fear cost trajectory σ=0.5

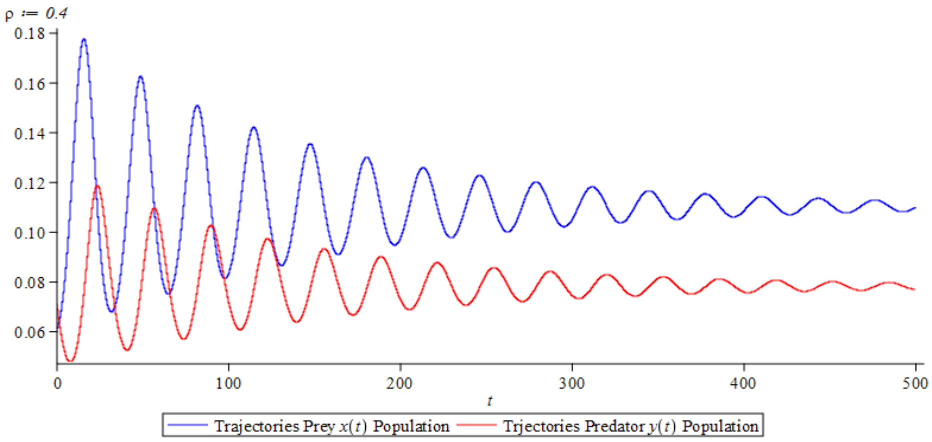

Figure 9. Fear cost trajectory σ=0.4

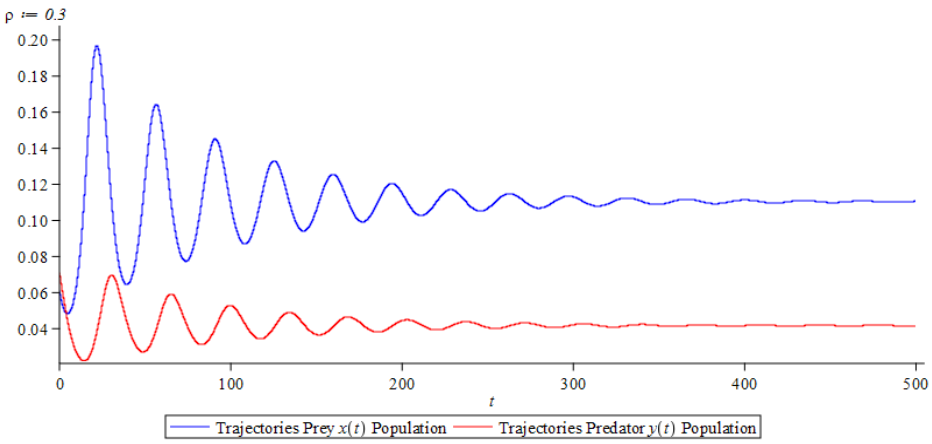

Figure 10. Fear cost trajectory σ=0.3

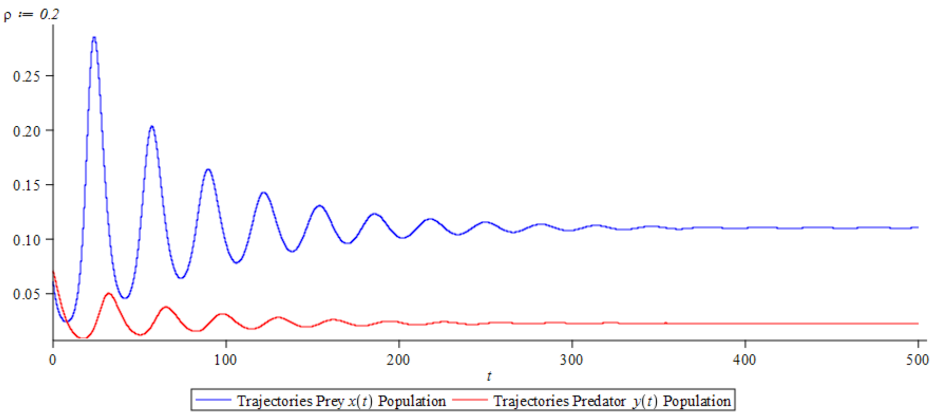

Figure 11. Fear cost trajectory σ=0.2

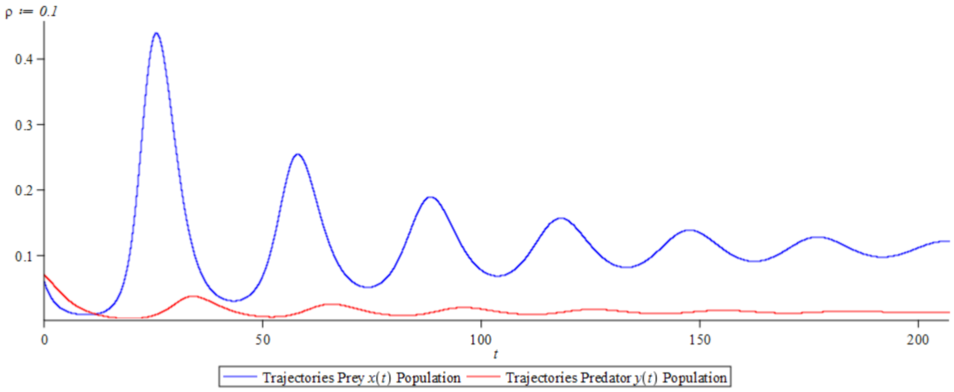

Figure 12. **Fear cost trajectory σ=0.2**

The fear cost trajectories (Figures 8 to 12) illustrate that the cost of fear dramatically affects the population growth of predator-prey species. The smaller the fear cost given by the simulation model in Equation 5, the more skewed the trajectory is. This suggests that when the cost of fear gets smaller, growth will lead to constant stability. On the other hand, the greater the cost of fear, the more volatile growth will lead to stability. The assumed parameter coefficients still consider the stability of the model. Parameters level and cost of fear in Equation 5 have maximum and minimum limits. In addition, it turns out that the cost of fear has specific symptoms of suppressing population growth. Low fear of prey, suppressing fluctuating predator population growth. In ecological conditions, this is realistic to consider. Meanwhile, the movement of prey population growth is relatively stable.

All parameters in Equation 5 provide a simple description of the actual ecological conditions. Realistic simulations are provided to estimate actual events in ecology. All parameters in the model have a significant effect, but the level and cost of fear are essential to population growth movement. Stability analysis in this model is also a consideration to determine the stability of the population. Stability analyzed through trajectories is very realistic in mathematical studies. The stability of the model and the trajectory indicate the survival of the species for a long time. Population extinctions and trajectories are not shown in Equation 5. However, what needs to be considered in the analysis of the analytical model is the very sharp level of fluctuation in certain conditions. Fluctuating sharp declines are expected to affect predator-prey populations significantly. At the level of fear, the predator population growth fluctuates with the increase in prey. This is inversely proportional to the cost of fear condition. The cost of fear tends to reduce the predator population growth rate significantly. The fluctuating movements between prey species and predator species significantly affect each other. Thus, it can be seen that population growth goes hand in hand.

4- Conclusion

Threatening behavior by predators dramatically affects the life behavior of prey because it provides a prolonged fear effect. The effect of fear on species is very realistic to consider because living things have fear. The fear effect responds to interactions that occur between predators and prey. The formulation of realistic assumptions is based on relevant previous research and realistic conditions that are very logical to accept. Variables that represent predator and prey species, respectively, are x and y, with all parameters that exist in the interaction model. Retrieval of parameter values based on assumptions and references. Every equilibrium that occurs is a solution of the differential equation. There is only one realistic biological and mathematical equilibrium point to consider, which is equilibrium point E_5 has a positive value and gives a negative eigenvalue in the characteristic equation. The characteristic equation gives an imaginary value with a negative real part that can be considered. The simulation model shows the survival conditions for the two species to continue to coexist. This strongly supports the concept of protecting species from extinction. The survival of each species is supported by ecological theory, which is against species extinction. In another mathematical approach, you can use the assumption of a symbiotic mutualism of species. Control is exerted on both species through coercion. Such control is necessary to sustain both populations in the long term. In both species, the numerical simulations show a local asymptotic balance. The visualization of the trajectory shows a stable fluctuation for a certain time. Prey species that experience stress from the threat effect of constant fear experience decreased sensitivity to threats. This change impacts the behavior of prey that dares to fight the effects of fear of predators, thereby affecting the growth rate of prey and the death rate of predators.

5- Declarations

5-1- Author Contributions

Conceptualization, R.A.P. and H.H.; methodology, R.A.P. and H.H; software, R.A.P. and H.H.; formal analysis, R.A.P. and H.H.; resources, R.A.P. and H.H.; data curation, R.A.P., M.L., and H.H.; writing—original draft preparation, R.A.P., and H.H.; writing—review and editing, R.A.P., H.H., W.R., and W.C.; supervision, W.C.; project administration, R.A.P., and H.H.; funding acquisition, R.A.P., and H.H. All authors have read and agreed to the published version of the manuscript.

5-2- Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5-3- Funding

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5-5- Institutional Review Board Statement

Not applicable.

5-6- Informed Consent Statement

Not applicable.

5-7- Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

6- References

[1] Márquez-Vera, M. A., López-Ortega, O., Ramos-Velasco, L. E., Rodríguez-Romero, A., Ramos-Fernández, J. C., & Hernández-Salazar, J. A. (2021). Adaptive threshold PCA for fault detection and isolation. Journal of Robotics and Control, 2(3), 119–125. doi:10.18196/jrc.2364.

[2] Lin, Z., Cui, C., & Wu, G. (2021). Dynamic modeling and torque feedforward based optimal fuzzy PD control of a high-speed parallel manipulator. Journal of Robotics and Control (JRC), 2(6), 527–538. doi:10.18196/jrc.26133.

[3] Purwono, P., Ma'arif, A., Mangku Negara, I. S., Rahmaniar, W., & Rahmawan, J. (2021). Linkage Detection of Features that Cause Stroke using Feyn Qlattice Machine Learning Model. Jurnal Ilmiah Teknik Elektro Komputer Dan Informatika, 7(3), 423. doi:10.26555/jiteki.v7i3.22237.

[4] Mukherjee, D. (2020). Role of fear in predator–prey system with intraspecific competition. Mathematics and Computers in Simulation, 177, 263–275. doi:10.1016/j.matcom.2020.04.025.

[5] Panday, P., Samanta, S., Pal, N., & Chattopadhyay, J. (2020). Delay induced multiple stability switch and chaos in a predator–prey model with fear effect. Mathematics and Computers in Simulation, 172, 134–158. doi:10.1016/j.matcom.2019.12.015.

[6] Sasmal, S. K., & Takeuchi, Y. (2020). Dynamics of a predator-prey system with fear and group defense. Journal of Mathematical Analysis and Applications, 481(1). doi:10.1016/j.jmaa.2019.123471.

[7] Sasmal, S. K., Kang, Y., & Chattopadhyay, J. (2015). Intra-specific competition in predator can promote the coexistence of an eco-epidemiological model with strong Allee effects in prey. BioSystems, 137, 34–44. doi:10.1016/j.biosystems.2015.09.003.

[8] Vinoth, S., Sivasamy, R., Sathiyanathan, K., Unyong, B., Rajchakit, G., Vadivel, R., & Gunasekaran, N. (2021). The dynamics of a Leslie type predator–prey model with fear and Allee effect. Advances in Difference Equations, 2021(1). doi:10.1186/s13662-021-03490-x.

[9] Xu, J., Zhang, T., & Han, M. (2019). A regime switching model for species subject to environmental noises and additive Allee effect. Physica A: Statistical Mechanics and Its Applications, 527. doi:10.1016/j.physa.2019.121300.

[10] Ang, T. K., & Safuan, H. M. (2019). Harvesting in a toxicated intraguild predator–prey fishery model with variable carrying capacity. Chaos, Solitons and Fractals, 126, 158–168. doi:10.1016/j.chaos.2019.06.004.

[11] Wang, J. S., Wu, Y. P., Li, L., & Sun, G. Q. (2020). Effect of mobility and predator switching on the dynamical behavior of a predator-prey model. Chaos, Solitons and Fractals, 132. doi:10.1016/j.chaos.2019.109584.

[12] Fakhry, N. H., & Naji, R. K. (2020). The dynamics of a square root prey-predator model with fear. Iraqi Journal of Science, 61(1), 139–146. doi:10.24996/ijs.2020.61.1.15.

[13] Liu, J., Lv, P., Liu, B., & Zhang, T. (2021). Dynamics of a Predator-Prey Model with Fear Effect and Time Delay. Complexity, 2021, 1–16. doi:10.1155/2021/9184193.

[14] Mondal, S., & Samanta, G. P. (2019). Dynamics of an additional food provided predator–prey system with prey refuge dependent on both species and constant harvest in predator. Physica A: Statistical Mechanics and Its Applications, 534. doi:10.1016/j.physa.2019.122301.

[15] Halder, S., Bhattacharyya, J., & Pal, S. (2022). Predator-prey interactions under fear effect and multiple foraging strategies. Discrete and Continuous Dynamical Systems - B, 27(7), 3779. doi:10.3934/dcdsb.2021206.

[16] Bajeux, N., & Ghosh, B. (2020). Stability switching and hydra effect in a predator–prey metapopulation model. BioSystems, 198. doi:10.1016/j.biosystems.2020.104255.

[17] Pal, S., Pal, N., Samanta, S., & Chattopadhyay, J. (2019). Effect of hunting cooperation and fear in a predator-prey model. Ecological Complexity, 39. doi:10.1016/j.ecocom.2019.100770.

[18] Tiwari, V., Tripathi, J.P., Mishra, S., & Upadhyay, R. K. (2020). Modeling the fear effect and stability of non-equilibrium patterns in mutually interfering predator–prey systems. Applied Mathematics and Computation, 371. doi:10.1016/j.amc.2019.124948.

[19] Dubey, B., Sajan, & Kumar, A. (2021). Stability switching and chaos in a multiple delayed prey–predator model with fear effect and anti-predator behavior. Mathematics and Computers in Simulation, 188, 164–192. doi:10.1016/j.matcom.2021.03.037.

[20] Lehtinen, S. O. (2021). Ecological and evolutionary consequences of predator-prey role reversal: Allee effect and catastrophic predator extinction. Journal of Theoretical Biology, 510. doi:10.1016/j.jtbi.2020.110542.

[21] Huang, Y., Zhu, Z., & Li, Z. (2020). Modeling the Allee effect and fear effect in predator–prey system incorporating a prey refuge. Advances in Difference Equations, 2020(1). doi:10.1186/s13662-020-02727-5.

[22] Roy, J., Barman, D., & Alam, S. (2020). Role of fear in a predator-prey system with ratio-dependent functional response in deterministic and stochastic environment. BioSystems, 197. doi:10.1016/j.biosystems.2020.104176.

[23] Qi, H., & Meng, X. (2021). Threshold behavior of a stochastic predator–prey system with prey refuge and fear effect. Applied Mathematics Letters, 113. doi:10.1016/j.aml.2020.106846.

[24] Mishra, P., & Tiwari, B. (2021). Drivers of pattern formation in a predator–prey model with defense in fearful prey. Nonlinear Dynamics, 105(3), 2811–2838. doi:10.1007/s11071-021-06719-2.

[25] Mukherjee, D. (2020). Study of fear mechanism in predator-prey system in the presence of competitor for the prey. Ecological Genetics and Genomics, 15. doi:10.1016/j.egg.2020.100052.

[26] Yousef, F. B., Yousef, A., & Maji, C. (2021). Effects of fear in a fractional-order predator-prey system with predator density-dependent prey mortality. Chaos, Solitons and Fractals, 145. doi:10.1016/j.chaos.2021.110711.

[27] Banerjee, R., Das, P., & Mukherjee, D. (2022). Effects of Fear and Anti-Predator Response in a Discrete System with Delay. Discrete and Continuous Dynamical Systems - Series B, 27(7), 3643–3661. doi:10.3934/dcdsb.2021200.

[28] Fardell, L. L., Pavey, C. R., & Dickman, C. R. (2020). Fear and stressing in predator–prey ecology: Considering the twin stressors of predators and people on mammals. PeerJ, 8. doi:10.7717/PEERJ.9104.

[29] Mondal, S., Samanta, G. P., & Nieto, J. J. (2021). Dynamics of a Predator-Prey Population in the Presence of Resource Subsidy under the Influence of Nonlinear Prey Refuge and Fear Effect. Complexity, 2021. doi:10.1155/2021/9963031.

[30] Datta, J., Jana, D., & Upadhyay, R. K. (2019). Bifurcation and bio-economic analysis of a prey-generalist predator model with Holling type IV functional response and nonlinear age-selective prey harvesting. Chaos, Solitons and Fractals, 122, 229–235. doi:10.1016/j.chaos.2019.02.010.

[31] Alsakaji, H. J., Kundu, S., & Rihan, F. A. (2021). Delay differential model of one-predator two-prey system with Monod-Haldane and holling type II functional responses. Applied Mathematics and Computation, 397. doi:10.1016/j.amc.2020.125919.

[32] Liu, X., & Huang, Q. (2020). Analysis of optimal harvesting of a predator-prey model with Holling type IV functional response. Ecological Complexity, 42. doi:10.1016/j.ecocom.2020.100816.

[33] Pratama, R. A., Toaha, S., & Kasbawati. (2019). Optimal harvesting and stability of predator prey model with Monod-Haldane predation response function and stage structure for predator. IOP Conference Series: Earth and Environmental Science, 279, 012015. doi:10.1088/1755-1315/279/1/012015.

[34] Sarkar, K., & Khajanchi, S. (2020). Impact of fear effect on the growth of prey in a predator-prey interaction model. Ecological Complexity, 42. doi:10.1016/j.ecocom.2020.100826.